Hedonic price methods and real estate price index: an explanatory study for apartments market in Belo Horizonte, Brazil from 2004 to 2015.

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Resumo: O Brasil ainda não possui um índice oficial de preço de imóveis residenciais. Todavia, após a crise do subprime, a demanda por índices de preços imobiliários aumentou. Em 2011, um decreto presidencial estipulou o Instituto Brasileiro de Geografia e Estatística a incumbência de criar e disseminar um índice de preços oficial para o mercado imobiliário brasileiro. Nesse artigo, são testados diferentes métodos hedônicos para estimar índices trimestrais para o mercado de apartamentos de Belo Horizonte, entre janeiro de 2005 e dezembro de 2015. A base de dados contempla o universo das transações com apartamentos no mercado formal de imóveis. Nossos objetivos são: i) estimar e comparar os diversos métodos hedônicos; ii) apresentar alguns resultados que irão contribuir para a discussão sobre o desenvolvimento de um índice de preços oficial para imóveis no Brasil. Os resultados corroboram com a conjuntura de intensa valorização imobiliária no Brasil entre 2007 e 2001, quando a valorização anual ficou acima dos 20%. Nos últimos anos da série – 2014 e 2015 – a taxa de crescimento diminuiu significativamente, abaixo dos 5%. Esses resultados mostram-se encorajadores no uso da metodologia hedônica e da base de dados de registros administrativos para a construção de índices de preços para imóveis no Brasil.

Palavras-chave: Índice de preços, Modelo de Preços Hedônicos, Mercado Imobiliário

Abstract: Brazil has not yet an official real estate price index. However, due to the great valuation of real estate prices in the last years and the 2008's subprime crisis, the demand for such indexes has increased. A 2011 presidential decree stipulated for the Brazilian Institute of Geography and Statistics (IBGE) the assignment to create and spread a real estate price index for Brazil. In this paper, we test some different hedonic model methods to estimate quarterly price indexes for apartments in Belo Horizonte, Brazil, from January 2005 to December 2015. The data set comprises all apartments transactions in the analyzed period from Belo Horizonte's real estate transmission tax. Our goals are: i) to measure and compare the different hedonic methods; ii) to present some results that will contribute to the discussion towards the development of an official real estate price index in Brazil. The empirical results corroborate the idea of intense apartment prices valuation in Belo Horizonte, mainly between 2007 and 2011, when the annual price growth taxes remained above 20%. However, in the two last years of the analyzed period, the annual growth tax has decreased below 5%. These results shed light on the potential use of both hedonic methods and administrative data base to construct an official real estate price index for Brazil.

Keywords: Price indices, Hedonic Price Model, Housing Market.

Classificação JEL: C43, E31, R31.

Área 1 – Teoria, métodos e modelos de economia regional

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1. -Introduction

Subprime crises turned attention around the world on the real estate price dynamics question. In Brazil the recent large valuation of real estate price adds more attention to the subject. Academics, news, real estate agents, Brazilians government agencies and statistical institutes started discussing the importance of having an adequate measure of real estate price over time. Federal Government Decree number 7.565 from 21 September, 2015, established IBGE (Brazilian Institute of Geography and Statistics) as the responsible to create and disseminate an official real estate price index for Brazil. Since then, IBGE and others governmental agencies have implemented studies regarding database and methodology to construct a future Brazilian official real estate price index (Nadalin and Furtado, 2011; Santos and Salazar, 2011).

The Brazilian academy has been studying real estate price index and its application in Brazilian context. In recent years, Rozenbaum (2009), Paixão (2015a) and Simões (2017) are examples of doctoral thesis related to this subject. Rozenbaum (2009) used administrative data to construct a hedonic quality adjusted price index for the city of Rio de Janeiro. Simões (2017) also measure the hedonic quality adjusted price index for Rio de Janeiro, using real estate agencies data, while Paixão (2015), also used administrative data to construct a hedonic quality adjusted price index for Belo Horizonte's city.

Some researches in this field were published in Brazilian academic journals. Gonzalez (1997) estimated a simple time-dummy hedonic model to construct price index for apartments rents in Porto Alegre. The same approach was used by Rozenbaum and Macedo-Soares (2007), to estimated real estate valuation in Rio de Janeiro's district of Barra da Tijuca, and by Paixão (2015a), to estimated real estate price indices for Belo Horizonte. Albuquerque *et al.* (2018) used repeated sales method to construct an index for the city of Brasilia, the capital of Brazil.

Some University's agencies like FIPE, from USP, and IPEAD, from UFMG, released real estate price indices using stratified median methods. The widespread FIPE-ZAP real estate index is calculated by FIPE from real estate's advertised data collected in ZAP's web site platform. The Brazilian Central Bank also estimated and published a monthly stratified median real estate price index, constructed from real estate loans data, called IVG-R. Despite the importance and relevance of those indices for the society, government, academics and real estate agencies some gaps still remain. None of those indices used the hedonic quality adjustment methodology, recognized as the best to deal with the nature of real estate's market data (Diewert, 2009; Hill *et al*, 2018). Besides that, only IPEAD uses administrative data which cover overall transacted dwellings market.

In this paper we try to construct quarterly hedonics quality adjusted real estate prices indices for Belo Horizonte's city using administrative data. For this task we will use the hedonics methods proposed by Hill's (2013) and Hill's *et al* (2018). We will use the same methods used by Hill *et al* (2018) in their analysis of Sidney and Tokyo markets, to produce comparable results.

The reminder of the paper is structured as follows. The next section explains the different hedonic price methods used to measure quality adjusted housing price indices. The third section is focused in the database and introduces the Brazilian's city of Belo Horizonte. The objective of the fourth section is to test the several hedonic methods in a Belo Horizonte's real estate market database. Finally, our main results are summarized in the conclusion.

2. Hedonic Quality Adjusted Real Estate Price Methods

2.1 – The Hedonic Price Model

Estimating housing price indices is a complex task. Housing is a type of complex good (or service), that is, a good where each unit or model differs from the others in qualitative terms. A complex good can be described as a bundle of many characteristics (or attributes), so each unit or model is a peculiar bundle of attributes. Computing index prices for complex good necessarily means controlling the change in the good price by the change in composition of its characteristics.

The hedonic price model establishes a functional relationship between the price of the good and its characteristics. In a hedonic perspective a good is a basket of characteristics Z, as represented below:

$$Z = Z\left(z_{1,} z_2, \ldots, z_n\right) \quad (1)$$

The price of a good follows a hedonic function as describes in (2):

$$P = P\left(z_{1,} z_2, \ldots, z_n\right) \quad (2)$$

Although only the price of the good can be observed in the market, the hedonic function establishes that the price of a good is determined by the composition of basket of characteristics. Therefore, each attribute (i) has an unobserved price (implicit price) that is represented by the first derivative of the hedonic function with respect to i.

$$p_i = \frac{\partial P}{\partial x_i} \quad (3)$$

The seminal paper of Rosen (1974) validated the hedonic price model in theoretical terms. Empirically, Waugh (1928) was pioneer in apply hedonic regression in vegetables market study. Court (1939) used hedonic price regression to construct automobile price indices. Griliches (1958, 1961) constructed hedonic quality adjusted price indices for fertilizers and automobile markets respectively. From Griliches contributions, the application of hedonic model widespread in the academic world, covering many types of different goods and services like computers, refrigerators, fruits, musical instruments, paints etc. However, it was in the real estate market that the hedonic approach achieved its largest projection.

2.2 – The Hedonic Quality Adjusted Price Indices: the real estate case

Griliches (1971) argues that a complex good's price change can be divided in two dimensions. The first is the observed price change of the good in the market. The second is the unobserved price change of the basket of characteristics. To estimate the unobserved price change, it is necessary to use the hedonic model regression as a quality adjusted factor. Discounting the price change of the attributes bundle from the observed price of the good results in a "pure" estimate of a complex good price change.

There are several ways to construct a quality adjusted price index from the hedonic methodology. Court (1939) and Griliches (1961) already advanced some questions, like the possibility of using both cross-section regressions or time-dummy approaches. Tripllet (2004) created a taxonomy of the several hedonic methods used to compute quality adjusted price indices for technological goods. Hill (2013) applied this Tripllest's taxonomy to the housing market case.

Hill *et al* (2018) using a Hill (2013) approach compiled the hedonic methods used by the European national statistics institutes. The first category embraces all indices which requires cross-section regressions models and, as a result, involves data imputation. The second category is based on time-dummy regressions.

2.2.1 – Imputation Approach

2.2.1.1 – Repricing Model

The first imputation method described by Hill *et al* (2018) is the repricing method. Like Hill *et al* (2018) in this study we adopted quarterly price indices as default and hedonic quality adjusted price indices could be constructed for any period. Defining the base period is the first task in the repricing model. Then a hedonic regression is estimated for this period. The price implicit estimated in hedonic regression is used to impute prices for each subsequent quarterly. The base period can be fixed or be updated at regular time intervals.

Hill *et al* (2018) recommends estimating one regression for the whole base year:

$$\ln(p)_{(1,q),h} = \sum_{c=1}^{C} \beta_{1,c} z_{(1,q),h,c} + \varepsilon_{(1,q),h} \quad (4)$$

Where $\ln(p)$ is the natural logarithmic of housing price in the base year (1), q is the quarterly of the sale, h is the dwelling sold and c is each characteristic of the dwelling.

Then the implicit prices estimated ($\hat{\beta}$) are used to estimated prices for each subsequent quarterly. The repricing method is, therefore, a sort of Laspeyres index. The quality adjustment factor $QAF_{(t,q-1),(t,q)}$ is defined as the ratio of the imputed prices for adjacent quarters, q and $q-1$ for example.

$$QAF_{(t,q-1),(t,q)} = \frac{\exp(\sum_{c=1}^{C} \hat{\beta}_{1,c} \bar{z}_{(t,q),c})}{\exp(\sum_{c=1}^{C} \hat{\beta}_{1,c} \bar{z}_{(t,q-1),c})} \quad (5)$$

To construct the repricing method price index (RP) a quality unadjusted price index ($QUPI_{(t,q),(tq-1)}$) defined as a ratio between geometric mean prices ($\tilde{p}$) of adjacent quarters is calculated as follows:

$$QUPI_{(t,q),(tq-1)} = \frac{\tilde{p}_{(t,q)}}{\tilde{p}_{(t,q-1)}} \quad (6)$$

Finally, RP is the ratio between quality unadjusted price factor and quality adjusted price factor.

$$\frac{P_{(t,q)}}{P_{(t,q-1)}} = \frac{QUPI_{(t,q),(t,q-1)}}{QAF_{(t,q),(t,q-1)}} \quad (7)$$

The main attractive feature of RP relies in the fact that it is not regression intensive. In the end, it requires only one regression (Hill *et al*, 2018). However, to achieve good results, Hill *et al* (2018, 224) suggested that "the base year under the repricing method should be updated at regular time intervals". Italy and Luxembourg national statistics institutes are benchmark examples since both updated the base every year (Hill *et al*, 2018).

2.2.1.2 – Average Characteristics Method

The average characteristic method (AC) requires, as any method, a definition of a base period. After that, the average characteristic of the dwellings sold in the base period are computed. The next step consists in estimating hedonic regressions for each subsequent period, quarterly in our case. Then the imputed prices are calculated, applying the estimated quarterly implicit prices on the average characteristics of the base period.

Following Hill *et al* (2018), the European national statistical institutes calculate de the basket of average characteristics for a whole year (base year), $\bar{z}_{t-1}$. In this line, the European national statistical institutes adopted a Laspeyres version of AC[1]. The base is updated every year. Then a hedonic regression is estimated for each quarter (q) of the following year (t):

$$\ln(p)_{(t,q),h} = \sum_{c=1}^{C} \beta_{t,c} z_{(t,q),h,c} + \varepsilon_{(t,q),h} \quad (8)$$

The quality adjusted price index estimated by AC is given as follows:

$$\frac{P_{(t,q)}}{P_{(t,q-1)}} = \frac{\exp(\sum_{c=1}^{C} \hat{\beta}_{(t,q),c} \bar{z}_{t-1,c})}{\exp(\sum_{c=1}^{C} \hat{\beta}_{(t,q-1),c} \bar{z}_{t-1,c})} \quad (9)$$

2.2.1.3 – Hedonic Imputation Method

Real estate is a threshold situation of complex goods. Each unity of real estate differs from the other. Added to this, the set of dwellings sale on one period differs of the set of dwellings sale in other periods. Therefore, it is not possible to construct a basket of dwellings to follow over time. The hedonic imputation method is a way to estimate the price each dwelling sold in t would have in another period, $t+1$ for example. According to Hill *et al* (2018, 225-6):

> Once a hedonic model has been estimated, it allows one to ask counterfactual questions such as what a particular dwelling actually sold in say period t would have sold for instead in period $t+1$.

Like the AC, one regression as (8) is estimated for each period. The regression in $t+1$ is used to impute the price in $t+1$ for each observed transacted dwelling in t. Likewise, the regression in t is used to impute the price in t for each observed transacted dwelling in $t+1$. To construct the index, Hill (2013) recommended to use the regression estimated price in t instead of the observed price for each observed transacted dwelling in t[2]. Such procedure is known as double imputation. From the hedonic imputation method geometric Laspeyres (GL), geometric Paasche (GP) and Tornqvist prices indices can be extracted.

Few European national statistical institutes use hedonic imputation methods. Hill *et al* (2018) follow the German version of double imputation Tornqvist (DIT). From a set of regressions like (8) the GL, GP and DIT are estimated as follows:

$$GL = \frac{exp \sum_{c=1}^{C} \hat{\beta}_{(t,q)} \bar{z}_{(t-1,c)}}{exp \sum_{c=1}^{C} \hat{\beta}_{(t,q-1)} \bar{z}_{(t-1,c)}} \quad (10)$$

$$GP = \frac{exp \sum_{c=1}^{C} \hat{\beta}_{(t,q)} \bar{z}_{(t,c)}}{exp \sum_{c=1}^{C} \hat{\beta}_{(t,q-1)} \bar{z}_{(t,c)}} \quad (11)$$

$$DIT = \sqrt{GL * GP} \qquad (12)$$

2.2.3. – Time-Dummy Approach

[2] Hill and Melser (2008) demonstrated in real estate case that double imputation is a way to minimize the omitted variable bias.

2.2.3.1 – Simple Time-Dummy

The time-dummy approach consists in constructing price indices from the estimated parameter of a dummy time variable in a hedonic regression. Usually, the first period in the series is used as the base. The simple time-dummy model (TD) requires only one regression and it is the simples and most intuitive hedonic method. The typical TD regression is illustrated as follows:

$$\ln(p)_{(t,q),h} = \sum_{c=1}^{C} \beta_{t,c} z_{(t,q),h,c} + \gamma_{t,q} D_{t,q} + \varepsilon_{(t,q),h} \quad (13)$$

Where $D_{t,q}$ is a time dummy for each period and $\gamma_{t,q}$ is the prince index estimated for the period q.

Despite its simplicity, there are some pitfalls in using TD (Hill, 2013). First, the TD does not allow the implicit price changes over time. As a result, the longer the series, the worst will the TD prince index estimations be. For national statistical institutes, TD is not recommended because it does not follow the temporal fixity criterium, as defined by Hill (2004). According this criterium, once an index has already been disseminated by the national statistical institute it should remain unchanged when new data becomes available. Using the single regression TD approach, when new data are added, a new estimation of (13) changes all parameters $\gamma_{t,q}$ previously disseminated.

2.2.3.2 – Rolling Time Dummy Method

The hedonic rolling time dummy method (RTD) consists in estimated hedonics time-dummy regressions for subperiods instead of only one regression for the whole period. The limiting case occurs when a regression is estimated for each pair of adjacent periods. Although when data points are scarce its recommended to estimate a regression including more than one subperiod.

France and Portugal, for example, estimated price index from an RTD with 2 quarter windows. In other words, both countries are using an adjacent period RTD. Other countries like Cyprus and Croatia estimated a 4 quarter windows RTD. The RTD price index is calculated from an RTD regression like (13) as showed below:

$$\frac{P_{q+1}}{P_q} = \frac{\exp(\hat{\gamma}_{q+1})}{\exp(\hat{\gamma}_q)} \quad (14)$$

3. The data

3.1 – Belo Horizonte: an overview

Belo Horizonte, the capital of the State of Minas Gerais, is an important economic, politic and cultural center in Brazil. According to 2010's Brazilian Census (IBGE, 2010), Belo Horizonte had a population of almost 2,4 million and was the 6th most populous city in Brazil. The Metropolitan Area (MA) of Belo Horizonte, in turn, had a population of 5,4 million and was the 3th most populous MA in Brazil.

Belo Horizonte was a planned city, conceived to replace Ouro Preto as the capital of Minas Gerais, and was founded in 1897. Nowadays, the planned area corresponds to downtown and its nearby districts bounded by Contorno Avenue. Like Aguiar *et al* (2014, 119) resumes:

This planning created a center-periphery radial model for the city, which concentrated urban services and urban infrastructure in particular areas, and reinforced social disparities.

From an administrative point of view, the space of Belo Horizonte is divided in Districts (487), Planning Units (82) and Regionals (9), see Figure A1 in appendix. Following Villaça (1998), historically, the Central-South city's Regional (Regional Centro-Sul) concentrated the elite's neighborhoods. Nowadays, some bordering Central-South Regional districts in the West Regional (Regional Oeste) are also occupied by Belo Horizonte's elite. Nonetheless,a few elite's districts are in Pampulha Regional (Regional Pampulha), in the north of the city, surrounding Pampulha's lagoon.

3.2 The Data

In Brazil all real estate transactions are subjected to The Real Estate Transfer Tax[3] (RETT) and its collection is in charge of municipalities. We used Belo Horizonte's municipality's RETT as our dataset, covering the 2004 to 2015 period, collected by IPEAD/UFMG. The RETT dataset contains the value of transaction, type of building, area, age, quality of building finishing material, zoning and location (district). The type of building includes apartments, houses and commercial real estate. In this paper we analyzed only apartments market. Further we shall expand the analysis for house and commercial real estate's markets.

Table 1 - Descriptive statistics for apartments in muicipality of Belo Horizonte: 2004-2015

Year	Observation	Value (reais)			Area (m2)			Age (years)		
		Mean	Median	Standard deviation	Mean	Median	Standard deviation	Mean	Median	Standard deviation
2004	17.767	90.682	60.122	91.427	122,0	102,9	70,6	12,4	8,0	11,7
2005	39.606	93.363	61.746	99.219	118,7	99,1	68,9	13,7	10,0	12,0
2006	37.614	107.572	70.196	117.326	120,4	102,1	69,2	14,3	10,0	12,2
2007	19.664	125.848	83.000	129.249	120,9	103,2	68,4	14,3	10,0	12,2
2008	19.224	155.117	100.000	156.457	121,4	101,7	70,6	14,5	10,0	12,5
2009	18.272	186.106	130.000	172.571	118,4	98,9	69,8	14,0	10,0	12,8
2010	21.177	227.110	160.764	201.441	114,4	94,4	68,5	12,1	8,0	13,1
2011	19.257	297.875	220.000	247.637	118,9	98,3	70,3	11,7	7,0	13,3
2012	18.408	362.735	274.412	284.955	123,8	107,0	70,9	11,8	6,0	13,5
2013	20.364	389.815	303.982	283.168	120,5	104,4	67,9	11,1	4,0	13,7
2014	19.516	418.895	330.000	291.744	120,6	105,7	66,9	10,1	2,0	13,3
2015	15.727	420.250	334.000	290.847	117,0	102,1	64,9	10,9	3,0	13,8
2004-2015	266.596	218.149	140.000	235.011	119,7	101,4	69,0	12,8	8,0	12,8

Sources: IPEAD/UFMG; author's calculation

Tables 1, 2 and 3 resume the data. There were 266.529 observations in our dataset for the whole period. The mean apartment price was around R$ 218.149 (approximately U$ 57 000) and the standard deviation was 235.011, indicating a high dispersion of this variable. Apartments sold in Belo Horizonte were fairy big and new, the mean area and age was 120 m^2 and 13 years, respectively. Most of apartments were classified as normal in terms of quality of building finishing material and this variable classified the quality in 5 categories. Ordering from the top there were the following categories: luxury, high, normal, low and popular. Most observations where located in the Center-South (Centro-Sul) and West (Oeste) Regionals.

[3] Imposto de Transmissão Imobiliária Inter-Vivos (ITBI).

TABLE 2 - Distribution of quality buiding finishing materials: Belo Horizonte, 2004-2015		
Quality of building finishing materials	Mean	Standard deviation
Popular	0,02	0,15
Low	0,21	0,41
Normal	0,59	0,49
High	0,16	0,37
Luxury	0,02	0,13
Sources: IPEAD/UFMG; author's calculation		

TABLE 3 - Apartaments transactions - mean for Regional - Belo Horizonte: 2004-2015		
Regional	Mean	Standard deviation
Centro-Sul	0,21	0,31
Leste	0,09	0,28
Nordeste	0,09	0,29
Noroeste	0,11	0,31
Oeste	0,20	0,40
Pampulha	0,15	0,35
Venda Nova	0,05	0,21
Barreiro	0,04	0,18
Norte	0,03	0,18
Sources: IPEAD/UFMG; author's calculation		

4 – Quality adjusted price indices for Belo Horizonte

4.1 – Model specification of the hedonics regressions

In section 2, we described the hedonic price model and its usefulness to design some quality adjust price indices for real estate's market. Since we adopted a log-linear estimation, the dependent variable was the natural logarithmic of the apartment's price. The set Z of independent variables was composed by numeric and dummies variables. The set of numeric variables was composed by area, age and squared age. It is expected that the higher is the apartment, the higher it will be the price. Age is a proxy of depreciation, so we expected a negative relationship between age and apartment's price. The effect of age, however, is not linear. Repairs and improvements reduce the depreciation age's effects. In addition, some old apartments are valued in the real estate market, the vintage effect phenomenon, which also reduces the age's negative effect in hedonic models (Goodman and Thibodeau, 1995).

The dummy variables set contains three subsets of variables. Apartment's physical conditions (quality of building finishing material) and apartment's location characteristics (zoning and UP location). Analyzing the quality of building finishing material, Table 2 shows that the worse category (popular) and superior category (luxury) represented a small share of the market. The most usual was the basic category (normal), thereby we expected a positive signal for luxury and high estimated parameters and negative signal for low and popular estimated parameters.

Zoning is an important apartment's price determinant. We chose as basic category ZAP (intensification use preferential zone), which corresponds to areas where the municipality encourages new buildings. ZPA and ZAR (restricted intensification uses zone) corresponds to areas where news buildings are not allowed or encouraged due to the natural or topographic conditions. Despite the new building restrictions, these areas are not valued in housing market because of its lack of affordable natural conditions. ZA (dense zone) represents high density areas where the municipality discourages new buildings. Commonly ZA's are in the most valued Belo Horizonte's districts and since it represents a supply restriction in a high demand context, we expected a positive estimated parameter on this variable. ZE (urban's infrastructure equipment zone) represents areas with great urban equipment (like bus stations). ZEIS (special social interest zone) correspond to original spontaneously occupation's areas (like informal slums), which were formalized by municipality. Both ZE and ZEIS represent poorly valued areas by the real estate's market agents, so we expected a negative signal for the estimated parameter on these variables. ZHIP (over central zone), ZCBH (Belo Horizonte's central zone), ZCVN (Venda Nova's central zone) and ZCBA (Barreiro's central zone) represent central areas. ZHIP is Belo Horizonte's downtown and ZCBH represent the other districts, besides Belo Horizonte's downtown - into Contorno Avenue boundary. Both represent valued land's locations – where firms and families wish to be located – consequently we expected a positive signal for estimated parameters on these variables. The central areas of Barreiro and Venda Nova's distant suburbs are represented by ZCBA and ZCVN. Since both zoning represent central areas in a minor magnitude, the same effect described above for ZCBH and ZHIP could occur.

As described in section 2, the construction of hedonic quality adjusted price indices is a regression intensive process. To illustrate the control variables included in the Z set of dependent variables we estimated a hedonic price model with quarter fixed effects. We estimated an ordinary least square (OLS) regression, as below:

$$\ln(p)_{t,q,h} = Z_{h,c}\beta_c + \gamma_{t,q}D_{t,q} + \varepsilon_{t,q,h} \qquad (15)$$

Where $\gamma_{t,q}$ represented quarter's fixed effect. The local fixed effects were contained on Z's characteristics set. The regression (15) corresponds to a simple time-dummy method. The results are expressed in tables 4, and 5.

Tabela 4 - Hedonic price model for Belo Horizonte's apartment market: 2004-2014

Belo Horizonte - 2004/2015

Variabel	Estimated parameter	Standard deviation	t	P-value
Constamt	10,49568	0,00514	2042,01	0,0000
Area	0,00664	0,00000942	705,04	0,0000
Age	-0,01833	0,00012124	-151,19	0,0000
Age^2	0,00015915	0,00000295	53,89	0,0000
Luxury	0,10088	0,00414	24,39	0,0000
High	0,05128	0,00148	34,59	0,0000
Low	-0,06656	0,00131	-50,69	0,0000
Popular	-0,09517	0,00369	-25,79	0,0000
ZPA	-0,053524	0,00857	-6,21	0,0000
ZAR	-0,05477	0,00162	-33,83	0,0000
ZA	0,13924	0,00231	60,23	0,0000
ZHIP	0,07185	0,00564	12,74	0,0000
ZCBH	0,44336	0,00359	123,34	0,0000
ZCVN	0,06234	0,0196	3,18	0,0015
ZCBA	0,19982	0,01279	15,62	0,0000
ZE	-0,17845	0,00704	-25,5	0,0000
ZEIS	-0,23219	0,0289	-10,61	0,0000
Spatial fixed effect = yes				
Time fixed effect = yes				
Ajusted R2	0,9236			
F	30399,1			0,0000
Sources: IPEAD/UFMG; authors calculation				

All estimated parameters had the expected signal and were significant at 5%. The marginal effect of area in apartments price was 0,67% which means that estimated shadow price of an additional m^2 for a mean price apartment was almost R$ 1.444. Each additional year depreciated the apartment in almost 1,8%. Nonetheless, due to squared age effect, each additional year added 0,016% on apartment value. Considering both age effects, the implicit price of each additional year on the mean price apartment was R$ 4.001. Quality of building finishing material classified as luxury added almost 10% on apartments price, while popular discounted around 10% on Belo Horizonte's apartment price.

Zoning was an important price's determinant. The affluent neighbor's downtown areas into Contorno Avenue's boundaries of ZCBH added 55,79% on apartments price. The low supply of vacant land combined with a large demand for both commercial and residential real estate caused this great valuation on ZCBH's apartment prices. In contrast, apartments located in ZEIS's area suffered a considerable discount (26,14%). As mentioned above these are areas with failed urban infrastructure, and most of them where slums originally. Finally, it is noteworthy the ZCBA's marginal effect, 22,12%, signaling a great demand for apartments in Barreiros's center.

The location dummy variables were all statistically significant. UP Savassi was the basic category. The signals were what we expected, except for the downtown's UPs of Barro Preto and Centro. Barro Preto, specifically, had the bigger negative marginal effect (-39,59%) among all. Part of this puzzling results should be the effect of the positive marginal effects of Barro Preto's zonings codes (ZHIP and ZCBH). In contrast, Belvedere, locate in the extreme south of Belo Horizonte, was the most valued UP, its price was, on average, 56,86% above the basic category.

4.2 – Quality adjusted price indices for Belo Horizonte

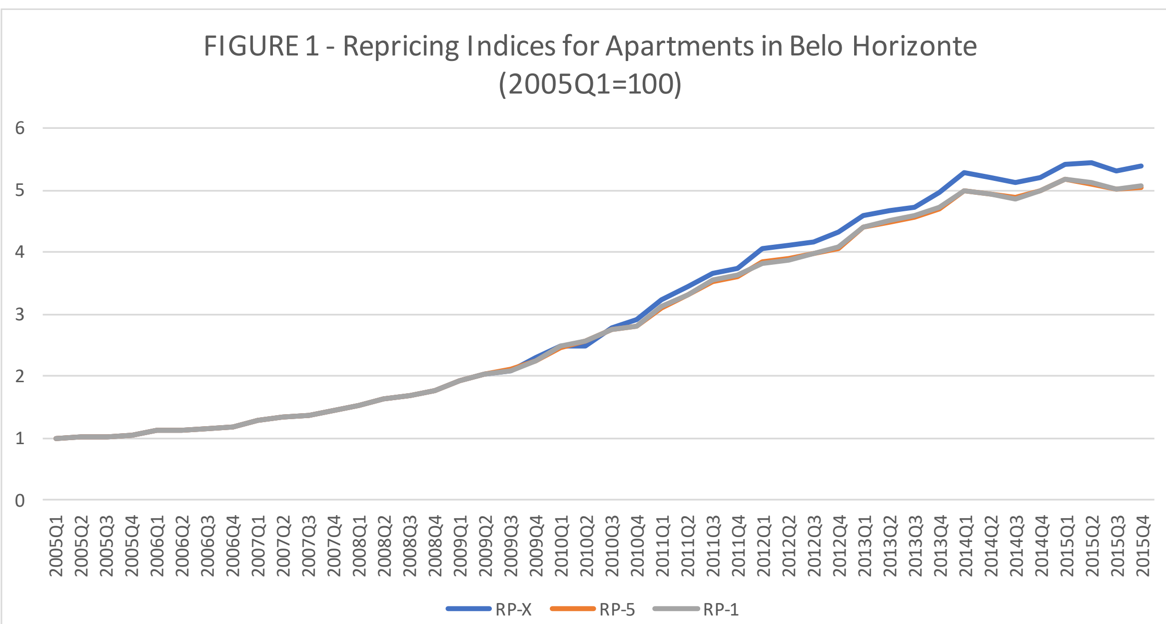
We estimated quarterly housing price indices for Belo Horizonte using the various methods discussed in sections 2.2.2 and 2.2.3 and an UP's stratified median method (MIX-UP). Since some methods were based on previous regression or mean characteristics, the 2004 data was used to compute reference baskets used in 2005. For this reason, results are represented for years 2005-2015. Following Hill *et al* (2018) we estimated three forms of repricing model (RP): i) (RP-X, which uses shadow price from 2004 (no updating base year); ii) RP-5, which updates the shadows prices every five years; iii) RP-1, which updates the shadows prices every year.

Table 5 - Estimated parameters for UP dummy				
Variable	Estimated parameter	Standard Deviation	t	P-value
Regional Leste				
Sagrada Família	-0,0264	0,0041	-6,3667	0,0000
Floresta	0,0171	0,0040	4,2710	0,0000
Pompéia	-0,1302	0,0071	-18,2411	0,0000
Santa Efigênia	-0,0689	0,0049	-14,1919	0,0000
Santa Inês	-0,0904	0,0069	-13,0247	0,0000
Regional Oeste				
Cabana	-0,3098	0,0066	-47,2627	0,0000
Jardim América	-0,1329	0,0037	-35,6164	0,0000
Barroca	0,0205	0,0033	6,1366	0,0000
Betânia	-0,2489	0,0053	-46,8326	0,0000
Buritis	0,0083	0,0035	2,3820	0,0172
Regional Centro-Sul				
Barro Preto	-0,5034	0,0062	-81,4118	0,0000
Centro	-0,1541	0,0051	-30,4518	0,0000
Prudente de Morais	0,1510	0,0043	34,7497	0,0000
Serra	0,0394	0,0043	9,1459	0,0000
São Bento	0,2052	0,0078	26,2196	0,0000
Belvedere	0,4503	0,0104	43,4269	0,0000
Anchieta	0,1415	0,0034	41,6524	0,0000
Regional Noroeste				
Glória	-0,3642	0,0057	-64,0957	0,0000
Padre Eustáquio	-0,0811	0,0040	-20,4647	0,0000
Camargos	-0,3228	0,0049	-66,1975	0,0000
PUC	-0,1169	0,0050	-23,2561	0,0000
Abílio Machado	-0,2440	0,0056	-43,2920	0,0000
Caiçara	-0,0622	0,0044	-14,1295	0,0000
Regional Pampulha				
Pampulha	-0,1864	0,0088	-21,0600	0,0000
Santa Amélia	-0,1895	0,0043	-43,9341	0,0000
Ouro Preto	-0,0931	0,0043	-21,6291	0,0000
Jaraguá	-0,1081	0,0041	-26,1436	0,0000
Castelo	-0,1394	0,0036	-38,2734	0,0000
Regional Nordeste				
Cachoerinha	-0,2334	0,0053	-44,2551	0,0000
Concórdia	-0,2021	0,0083	-24,4453	0,0000
Cristiano Machado	-0,0216	0,0034	-6,3133	0,0000
São Paulo	-0,3012	0,0048	-62,5666	0,0000
Regional Norte				
Planalto	-0,2793	0,0054	-51,8408	0,0000
São Bernardo	-0,3022	0,0065	-46,2063	0,0000
Primeiro de Maio	-0,2980	0,0091	-32,7033	0,0000
Jaqueline/Tupi	-0,4589	0,0056	-82,5165	0,0000
Regional Barreiro				
Barreiro de Baixo	-0,3479	0,0045	-77,3910	0,0000
Cardoso	-0,4978	0,0057	-87,8971	0,0000
Regional Venda Nova				
Europa	-0,3088	0,0059	-52,3095	0,0000
Venda Nova	-0,2741	0,0043	-63,3277	0,0000
Source: IPEAD/UFMG; author's calculation				

The average characteristic indexes (AC) were estimated with a base update every year. In AC's case, the base is the one year lagged average characteristics. The double imputation indexes were calculated estimating, for some quarter set of observations, counterfactual housing's basket for previous and posterior periods. The double imputation Laspeyres (DIT), Paasche (DIP) and Tornqvist (DIT) were estimated as presented in section 2.2.1.3. The rolling time dummy indices were estimated for 2 (RTD2) and 4 (RTD4) quarters window. Finally, we estimated an UP's stratified median (MIX-UP) index, to compare the quality adjusted hedonic housing price index with a simpler and more intuitive price index. Table 6 resumes the quarter housing price indices estimated.

From the chosen period (2005-2015), the large appreciation of housing prices in Brazil was supported by estimated indices. Even so, the magnitude of the appreciation differs between methods. Using the DIP index, Belo Horizonte's house prices rose 383,6% in contrast with DIL which shows an increase of 435,7%.

RIP-X, which uses shadow price form 2004, was apartheid from RP-5, which changes the base every five years, and RP-1, which updates shadow prices every year (Figure 1). From our Belo Horizonte's database, the RIP-X seems not be an appropriate index due to its fail to control shadow prices change over time. RP-5 and RP-1 lines were close each other, highlighting the importance to update the base year from time to time. Since RP-1 is the more flexible RP index it will be used in the remainder of the paper.



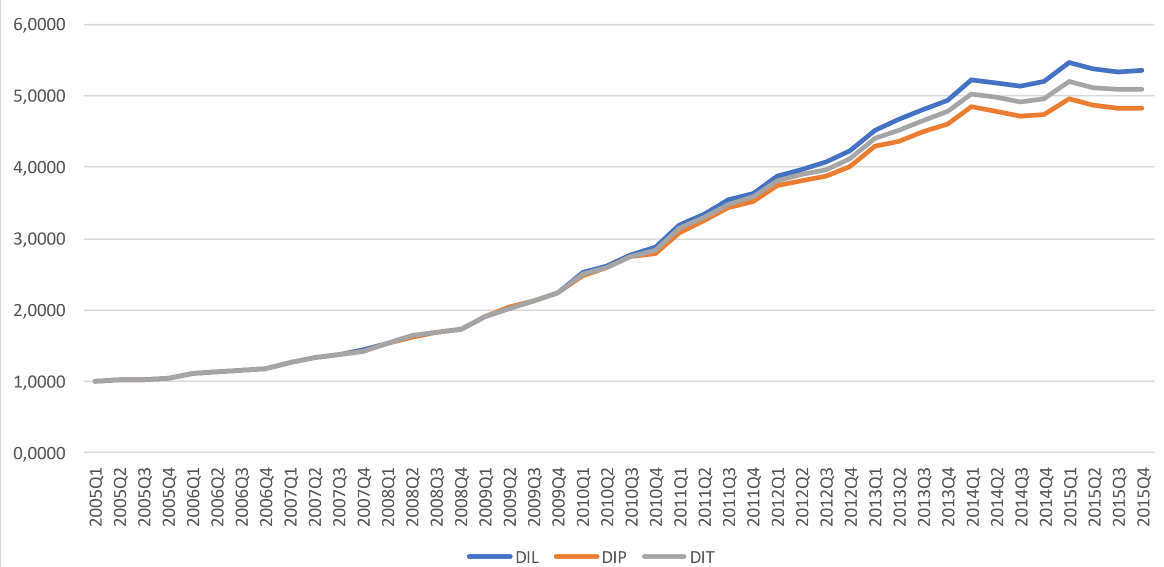
Sources: IPEAD/UFMG; author's calculation

The double imputation Laspeyres (DIL) and double imputation Paasche (DIP) showed evidence of drift (Figure 2), as Hill *el al* (2018) also have noted for Sydney data. For Belo Horizonte apartment's market, DIP estimated the smallest price variation while DIL estimated the biggest. In agreement with price index theory, DIL, as a Laspeyres index, tends to overestimate the price change and DIP, as a Paasche index, tends to underestimate it. Double imputation Tornqvist (DIT), in any case, does not exhibit a drift. DIT, as a Tornqvist-Geometric index, is a geometric mean of DIP and DIL. Bearing in mind that the Tornqvist indices are recognized as superlatives, the DIT becomes an attractive alternative method to compute housing indexs prices. Since DIP a DIL exhibited a drift behavior, the imputation methods will be reduced to DIT in the following analysis.

The rolling time dummies (RT) were estimated for 4 quarter window (RT4) and 2 quarter window (RT2) from our database. On the RT4 index the quarter base changes once a year, and on RT2 the base changes every quarter. RTs methods are attractive because the index corresponds to the estimated regression time dummy parameter. RT2 indices stayed above RT4 for the whole period. Both RT indices will be kept in the following analyses.

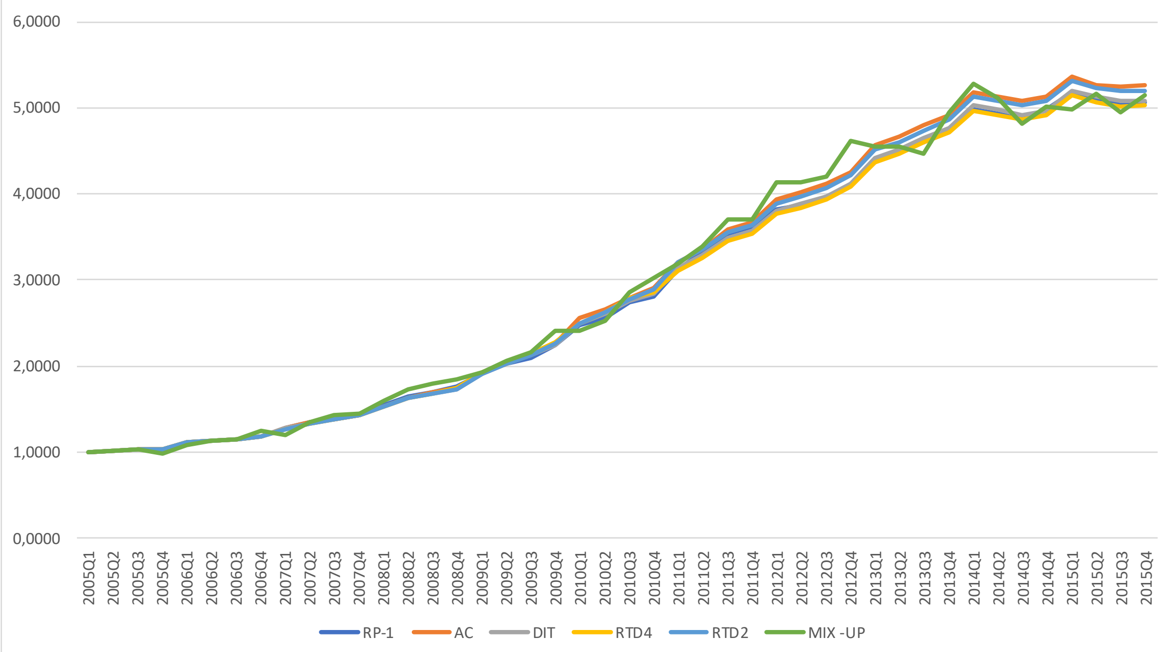
The Figure 3 compares the Belo Horizonte's hosing price indices estimated by different hedonic methods and by stratified median.

FIGURE 2 - Double Imputation Indices for Apartments in Belo Horizonte (2005Q1=100)



Source: IPEAD/UFMG, author's calculation

Figure 3 - Estimate of Price Indices for Apartments in Belo Horizonte (2005Q1=100)



Source: IPEAD/UFMG, author's calculation

DIT, RP1 and RT4 exhibited a very close behavior. AC and RT2 do the same, although the latter stayed above the former in the most recent quarters. Partly, the different behavior between AC and DIT is expected, since the first is a Laspeyres type of index and the second is a Tornqvist type. The MIX-UP line was more volatile than the other indicex lines due to the lack of characteristics control related to this method.

Hill *et al* (2018) recommended analyzing the volatility of indices in more details. The authors present two volatility measures: the root mean squared error (RMSE) and mean absolute deviation (MAD). Also, they calculated the minimum (MIN) and maximum (MAX) value for each index. All these indicators are computed both on a year-by-year and quarter-by-quarter basis. The indicators volatility formulas are specified below:

$$RMSE = \sqrt{\frac{1}{T-1}\sum_{t-1}^{T-1}\left[ln\left(\frac{p_{(t+1,q)}}{p_{(t,q)}}\right) - \frac{1}{T-1}\left(\frac{p_{(T,q)}}{p_{(1,q)}}\right)\right]} \qquad (15)$$

$$MAD = \frac{1}{T-1}\sum_{t-1}^{T-1}\left|ln\left(\frac{p_{(t+1,q)}}{p_{(t,q)}}\right) - \frac{1}{T-1}ln\left(\frac{p_{(T,q)}}{p_{(1,q)}}\right)\right| \qquad (16)$$

$$MIN = Min_{1,\dots,T-1}\left\{100\left[\frac{p_{(t+1,q)}}{p_{(t,q)}}\right] - 1\right\} \qquad (17)$$

$$MAX = Max_{1,\dots,T-1}\left\{100\left[\frac{p_{(t+1,q)}}{p_{(t,q)}}\right] - 1\right\} \qquad (18)$$

The results are summarized in a Table 7.

Table 7 - Volatily of The House Price Indices in Belo Horizonte

	RP-X	RP-5	RP-1	AC	DIL	DIP	DIT	RTD4	RTD2	MIX-UP
Year-on-Year (Q1)										
RMSE	0,0723	0,0642	0,0628	0,0678	0,0636	0,0654	0,0642	0,0633	0,0667	0,1032
MAD	0,0629	0,0569	0,0551	0,0575	0,0524	0,0559	0,0541	0,0537	0,0564	0,0869
MIN	2,4779	3,6662	4,0808	3,6532	4,7136	2,4382	3,5696	3,3962	3,5155	-5,6054
MAX	31,0888	28,5716	29,0456	32,4961	32,0324	29,2254	30,6214	29,6188	30,0129	32,8800
Year-on-Year (Q2)										
RMSE	0,0723	0,0656	0,0649	0,0684	0,0649	0,0664	0,0655	0,0647	0,0681	0,0777
MAD	0,0558	0,0533	0,0530	0,0559	0,0506	0,0548	0,0527	0,0515	0,0542	0,0632
MIN	2,4779	3,6662	4,0808	3,6532	4,7136	2,4382	3,5696	3,3962	3,5155	-5,6054
MAX	31,0888	28,5716	29,0456	32,4961	32,0324	29,2254	30,6214	29,6188	30,0129	32,8800
Year-on-Year (Q3)										
RMSE	0,0773	0,0739	0,0757	0,0750	0,0715	0,0739	0,0726	0,0723	0,0751	0,0846
MAD	0,0668	0,0651	0,0656	0,0636	0,0594	0,0635	0,0614	0,0608	0,0638	0,0766
MIN	3,6008	2,6512	3,4586	3,3906	4,0176	2,5779	3,2952	3,2302	3,2725	2,8609
MAX	33,3710	29,0503	31,0202	30,4117	30,4740	29,0106	29,7403	30,0454	30,6093	32,7789
Year-on-Year (Q4)										
RMSE	0,0753	0,0750	0,0761	0,0774	0,0748	0,0769	0,0757	0,0747	0,0776	0,0896
MAD	0,0654	0,0651	0,0661	0,0653	0,0616	0,0655	0,0635	0,0622	0,0653	0,0800
MIN	3,9608	1,2501	1,5353	2,7994	2,8468	1,9609	2,4029	2,3601	2,1151	1,3200
MAX	29,1409	28,1510	28,6306	29,7209	29,7547	29,0226	29,3882	30,1850	30,1020	30,5135
Quater-on-Quarter										
RMSE	0,0326	0,0294	0,0291	0,0306	0,0289	0,0294	0,0290	0,0283	0,0294	0,0470
MAD	0,0277	0,0253	0,0243	0,0243	0,0233	0,0243	0,0237	0,0237	0,0242	0,0388
MIN	-2,6650	-1,7031	-1,7031	-1,7786	-1,5315	-1,5806	-1,5560	-1,5670	-1,6609	-5,8666
MAX	11,9847	10,6583	10,6583	13,3085	11,8749	10,4981	11,1844	10,4400	11,0705	13,2288

Sources: IPEAD/UFMG; author's calculation

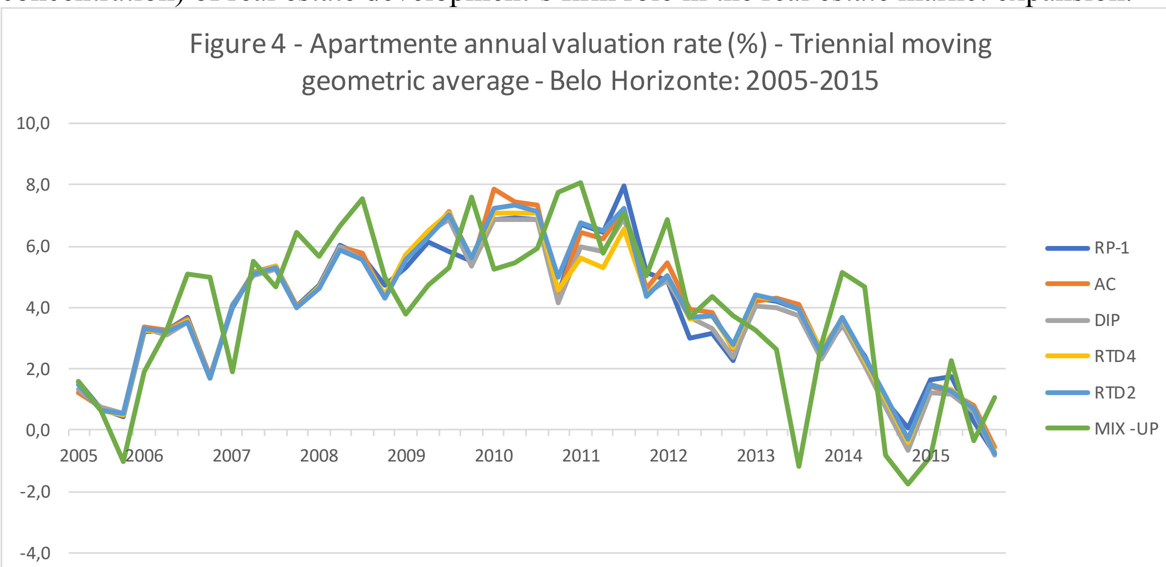
MIX-UP is more volatile than the other indices. As Hill *et al* (2018) pointed out, it is expected for stratified median indices (like MIX-UP) to exhibit more volatility since they are not adjusted for changes in the quality of median over time. The hedonics quality adjusted indices exhibited relative low volatility, the magnitudes were between those which were estimated for Sidney and Tokyo by Hill *et al* (2018). From volatility indicators perspectives our results suggest, for Belo Horizonte's housing market in 2005-2015, that the hedonic quality adjusted housing price indices were accurate, except for the MIX-UP cases.

4.3 – Apartment Valuation in Belo Horizonte: 2005-2015

We will illustrate the previous results measuring quarterly apartments prices rate of appreciation for Belo Horizonte. Table 8 summarizes the results for different methodologies.

Table 8 - Rate of appreciation (%) of apartment prices in Belo Horizonte: 2005-2015							
Year	Quarter	RP-1	AC	DIP	RTD4	RTD2	MIX -UP
2005	Q2	1,59	1,22	1,32	1,51	1,48	1,60
2005	Q3	1,39	1,50	1,52	1,31	1,30	1,32
2005	Q4	0,59	0,85	0,92	0,88	0,97	-3,70
2006	Q1	7,86	7,86	7,65	7,73	7,75	8,52
2006	Q2	1,43	1,16	0,95	1,16	1,10	5,40
2006	Q3	1,84	2,04	2,03	1,98	1,90	1,53
2006	Q4	2,00	2,00	2,09	1,99	2,06	8,09
2007	Q1	8,33	8,25	8,25	8,30	8,24	-3,55
2007	Q2	5,14	5,23	4,95	5,08	5,07	12,59
2007	Q3	2,66	2,63	2,63	2,71	2,74	5,59
2007	Q4	4,33	4,27	4,37	4,24	4,15	1,49
2008	Q1	7,19	7,07	7,16	7,09	6,96	10,12
2008	Q2	6,64	6,65	6,44	6,28	6,56	8,50
2008	Q3	3,24	3,65	3,19	3,35	3,26	4,09
2008	Q4	4,34	2,71	3,33	3,57	3,12	2,53
2009	Q1	8,39	10,94	10,32	10,44	10,44	4,73
2009	Q2	5,66	5,59	5,80	5,57	5,49	7,02
2009	Q3	3,55	5,00	4,57	5,37	5,20	4,25
2009	Q4	7,31	5,47	5,70	5,97	6,15	11,70
2010	Q1	9,91	13,31	10,50	9,96	10,36	0,09
2010	Q2	3,68	3,87	4,53	5,45	5,62	4,89
2010	Q3	7,14	5,05	5,66	5,84	5,56	13,23
2010	Q4	2,46	4,13	2,29	2,38	3,85	5,38
2011	Q1	10,66	10,37	10,12	8,78	11,07	5,86
2011	Q2	6,45	4,35	5,27	4,87	4,76	6,13
2011	Q3	6,80	6,87	5,53	6,01	6,06	9,26
2011	Q4	2,25	2,63	2,69	2,38	2,34	0,02
2012	Q1	5,61	6,92	6,47	6,85	6,73	11,67
2012	Q2	1,19	2,40	1,88	1,70	2,04	-0,25
2012	Q3	2,75	2,21	1,68	2,66	2,49	1,97
2012	Q4	2,95	3,31	3,53	3,78	3,79	9,78
2013	Q1	7,30	7,24	6,97	6,67	6,96	-1,59
2013	Q2	2,44	2,46	1,63	2,32	2,01	0,02
2013	Q3	2,10	2,70	2,74	2,94	2,88	-1,91
2013	Q4	2,85	2,56	2,59	2,61	2,56	10,75
2014	Q1	5,39	5,14	5,16	5,38	5,61	6,94
2014	Q2	-0,92	-0,73	-1,23	-1,08	-0,95	-3,14
2014	Q3	-1,46	-1,12	-1,48	-1,14	-1,13	-5,87
2014	Q4	2,57	0,96	0,69	1,09	1,15	3,91
2015	Q1	3,93	4,60	4,55	4,59	4,50	-0,37
2015	Q2	-1,27	-1,78	-1,58	-1,57	-1,66	3,38
2015	Q3	-1,70	-0,32	-1,00	-0,81	-0,64	-3,90
2015	Q4	0,67	0,38	0,09	0,24	0,01	3,91
Sources: IPEAD/UFMG, author's calculation							

From the first quarter of 2007 to the first quarter of 2014 there was an intense apartment's price appreciation in Belo Horizonte. This situation agrees with Brazilian real estate outlook. This appreciation was contemporary with the expansion rate of housing credit in Brazil. Some institutional improvements like fiduciary alienation law's refinement in 2004 agreed with income growth and the decline interest rates helped the housing credit's growth (Aguiar, 2014). Cardoso and Leal (2009) highlighted the government politics and the restructuring (more market concentration) of real estate development's firm role in the real estate market expansion.



Source: IPEAD/UFMG, author's calculation

Figure 4 shows the quarterly variation of Belo Horizonte's apartments prices. It's clear in the figure the great volatility of the median index (MIX-UP), as we have seen in the previous section. From 2005 to 2011 there was a significant housing prices growth path, from then there was a decline tendency. After the second quarter of 2014, the decline was more intense due to the Brazil's economic crises which began in this period.

Conclusion

Brazil does not have an official price index yet. In this paper we used a database from Belo Horizonte, a big Brazilian city, to test some hedonic quality adjusted price indices. The indices constructed were the same used by European Statistical Institutes as described by Hill *et al* (2018). Our results suggested that the hedonic quality adjusted indices exhibited a good performance in volatility terms. However, it was detected some drift in double imputation Laspeyres and Paasche indices. The former with a strong upper-ward bias relative to the other hedonic indices and the latter with a strong down-ward bias.

From our analyses, the double imputation Tornqvis (DIT) and the repricing with an annual base's update (RP-1) produced very similar magnitude's indices. The same could be said about the average characteristics (AC) and the 2-quarter rolling time dummy (RT2). However, the index price lines of the latter stayed above the former. The 4-quarter rolling time window, in its turn, exhibited an intermediate behavior, as compared with the previously listed indices.

In contrast to Hill *et al* (2018) estimation for Sidney and Tokyo's evidence, the repricing method with no base update (RP-X) had an upper-ward bias relative to the other hedonic indices. The five years update base repricing method, as well as Hill *et al* (2018) empirical evidences, exhibited a down-ward bias. Finally, our models result suggested that median indices are not the most appropriate to estimate housing price indices. The stratified median index (MIX-UP) used was more volatile than the hedonic indices. This is because this kind of index is imperfect in control for housing quality variation over time.

This paper emphasized the potentiality for constructed housing price indices in Brazil using hedonic quality adjusted price methods and for administrative data. The Real Estate Transfer Tax (RETT) emerges as a hopeful database once this tax is collected in the whole country. Further analysis could extend the hedonic price models to estimate price indices for the Brazilian smaller city context and its less frequent housing sales reality. The smaller number of observations imposes new challenges to estimated housing prices hedonic quality adjusted indices. In addition, future analysis could extend the types of real estate units, including houses and different types of commercial real estates.

References

AGUIAR, Marina Moreira de, SIMÕES, Rodrigo; GOLGHER, André Braz. Housing Market Analysis using a hierarchical-spatial approach: the case of Belo Horizonte, Minas Gerais, Brazil. **Regional Studies**, Regional Science, v.1, n.1, 2014, p. 116-137.

AGUIAR, Marina Moreira de. Crédito e mercado imobiliário no Brasil: fatores institucionais e macroeconômicos (1964-2013). XVI SEMINÁRIO DE ECONOMIA MINEIRA, Diamantina, 2014. **Anais.**.. Diamantina: CEDEPLAR/UFMG, 2014.

ALBUQUERQUE, Pedro; NADALIN, Vanessa; NETO, Vicente and MONTENEGRO, Mariana. Construção de índices de preços de imóveis para o Distrito Federal por meio de vendas repetidas e GWR. **Nova Economia**, v.28, n.1, p 181-212.

CARDOSO, Adauto Lúcio; LEAL, José Agostinho. Housing Markets in Brazil: recent trends and governmental responses to the 2008 crisis. ISA INTERNATIONAL HOUSING CONFERENCE, Glasgow, 2009. Anais… Glasgow: ISA, 2009.

COURT, Andrew. Hedonic price indexes with automotive examples. **The Dynamic of Automobile Demand**: General Motors, New York, p.98-119, 1939.

DIEWERT, W. Erwin. The Paris OECD-IMF Workshop on Real Estate Price Indexes: conclusions and future directions. In: DIEWERT, E. Erwin *et al* (Ed.). **Price and Productivity Measurement:** volume 1 – housing. Trafford Press, 2009.

GONZÁLEZ, Marco. Variação qualitativa e índice de preços na análise do comportamento recente dos aluguéis em Porto Alegre (1994-1997). **Análise Econômica**, v.15, n.28, 1997.

GOODMAN, Allen; THIBODEAU, Thomas. Age-related heteroskedasticity in hedonic price equations. **Journal of Housing Research**, v. 6, n. 1, p. 25-42, 1995.

GRILICHES, Zvi. Hedonic Price indexes for automobiles: an econometric of quality change. **In: The Price Statistics of the Federal Government**, General Series No. 73. Columbia Univ. Press for the National Bureau of Economic Research, New York, p. 137-196, 1961.

GRILICHES, Zvi. Introduction: hedonic price indexes revisited. In: **_____** (Ed.). **Prices indexes and quality change**: studies in new methods of measurment. Cambridge: Harvard University Press, 1971.

GRILICHES, ZVI. The Demand for fertilizer: an econometric reinterpretation of a technical change. **Journal of Farm Economics**, v. 40, n. 3, p.591-606, 1958.

HILL, Robert and MELSER, Daniel. Hedonic imputation and the price index problem: an application to housing. **Economic Inquiry**, v. 46, n. 4, p. 593-609, 2008.

HILL, Robert; SCHOLZ, Michael; SHIMIZU, Chihiro and STEURER, Mirian. An evaluation of the methods countries to compute their official house prices indices. **Economie et Statistique/ Economics and Statistics**, v. 500-501-502, p. 221-238, 2018.

HILL, Robert. Constructing price indexes across space and time: the case of the European Union. **American Economic Review**, v.04, n. 5, p. 1379-1410, 2004.

HILL, Robert Price indexes for residential housing: a survey, evaluation and taxonomy. **Journal of Economic Surveys**, v.27, n.1, p. 879-914, 2013;

NADALIN, Vanessa and FURTADO, Bernardo. Índices de preços para imóveis: uma revisão. **Boletim Regional, Urbano e Ambiental**, n.6., p.21-32, 2011.

PAIXÃO, Luiz. **Índice de preços hedônicos para apartamentos:** Belo Horizonte, 1995-2012. Tese (Doutorado em Economia). Programa de Pós-Graduação em Economia. Universidade Federal do Rio de Janeiro, 2015.

PAIXÃO, Luiz. Índice de preços hedônicos para imóveis: uma análise para o município de Belo Horizonte. **Economia Aplicada**, v.19, n.1, p.5-29, 2015a.

ROSEN, Sherwin. Hedonic price and implicit markets: products differentiation in pure competition. **Journal of Political Economy**, v.82, n.1, p.35-55, 1974.

ROZENBAUM, Sérgio and MACEDO-SOARES, T. Diana. Proposta para a construção de um índice de preços de imóveis a partir dos lançamentos imobiliários de condomínios residenciais. **Revista de Administração Pública**, v.41, n.6, 2007.

ROZENBAUM, Sérgio. **IMPA – Índice municipal de preço de apartamentos:** proposta e metodologia. Tese (Doutorado em Administração de Empresas). Programa de Pós-Graduação em Administração de Empresas. Pontifícia Universidade Católica do Rio de Janeiro, 2009.

SANTOS, Flávia and SALAZAR, Marlon. **Índice de preços imobiliários para o Brasil**: estudos para discussão. Rio de Janeiro: Instituto Brasileiro de Geografia e Estatística (IBGE), 2011.

SIMÕES, Paulo Fernando Mahaz. **Modelos longitudinais de efeitos mistos:** aplicações ao cálculo de índices de preços de imóveis e à análise da perda não técnica de eletricidade. Tese (Doutorado em Engenharia Elétrica). Programa de Pós-Graduação em Engenharia Elétrica. Pontifícia Universidade Católica do Rio de Janeiro, 2017.

TRIPLETT, Jack. **Handbook on hedonic and quality adjustments in price indexes**: special application to information technology products. Paris: Organization for Economic Co-operation and Development (OECDE), 2004. (Working Paper 2004/9)

VILLAÇA, Flávio. **O espaço intraurbano no Brasil**. São Paulo: Studio Nobel, 1988.

WAUGH, Frederik. Quality factors influencing vegetable prices. **Journal of Farm Economics**, v.10, n.2, p.185-196, 1928.