

Cournot-Bertrand Cartel Stability on Antitrust Enforcement*

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Resumo

Dado um acordo colusivo entre duas empresas em um modelo de duopólio padrão em que os produtos são diferenciados horizontalmente, usamos uma estrutura de modelo de Bertrand-Cournot para mostrar que a empresa que escolhe quantidades desconta muito o futuro do que a empresa que escolhe preços para todos os graus de diferenciação. A empresa que escolhe preços é mais paciente, com um fator de desconto menor. Também demonstramos que a aplicação de leis antitruste desestabiliza fortemente o acordo colusivo anterior para empresas de produtos mais diferenciados. O resultado contribui para indicar quais mercados potencialmente precisam de uma ação antitruste mais forte.

Palavras-chave: Cartel Cournot-Bertrand, fiscalização antitruste, estabilidade de cartel.

Abstract

Given a collusive agreement between two firms in a standard duopoly model where products are horizontally differentiated, we use a Bertrand-Cournot model framework to show that the firm that chooses quantities discounts the future heavily than the firm that chooses prices for all degrees of differentiation. The firm that chooses prices is more patient, with a smaller discount factor. We also demonstrate that antitrust enforcement destabilizes the previous collusive agreement strongly for more differentiated products firms. The result contributes to indicate which markets potentially need stronger antitrust action.

Keywords: Cournot-Bertrand cartel, enforcement antitrust, cartel stability.
JEL: D4, C23, L91, R32.

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1 Introduction

Cartels¹ are considered a type of organized crime that threatens the economic order because it allows responsible firms to benefit from the absence of competition. The misallocation of society resources is due to higher prices paid by consumers, reduction in output, quality, and product diversification. To minimize these problems, the antitrust authority can directly increase economic efficiency by legally processing existing cartels. And indirectly, decreasing the appearance of new cartels and destabilizing formed ones.

Collusive agreements are usually actions to combine prices and/or market shares (including allocation of specific territories for parties). There are several reasons for cartels instability, the possibility of cheating expanding output is one of those. The level of indebtedness of firms, trade liberalization, and demand volatility can also increase uncertainty, and by consequence, the cartels' instability. These reasons are pointed out by [Levenstein and Suslow \(2011\)](#) as some of the factors that increase the probability of "natural death" of the cartels. Another possibility is the instability caused by antitrust intervention.

Degree of product differentiation also plays a role in cartels stability. Among those who studied the effects of the degree of differentiation on the stability of the cartel were: [De-neckere \(1983\)](#), in a standard model of duopoly differentiated horizontally, says that the agreements by price are more tacit than by quantity when goods are complements or very close substitutes. [Chang \(1991\)](#), in a Hotelling model, indicates that the ability to collude decreases when products become more homogeneous. [Ross \(1992\)](#), also in particular demonstrates that a higher degree of product homogeneity can reduce the stability of the cartel. On the other hand, the presence of the antitrust authority is also another factor that destabilizes them. Among those who studied cartel dynamics in the presence of antitrust authority are [Motta and Polo \(2003\)](#), [Houba et al. \(2015\)](#), and [Harrington Jr \(2017\)](#), without emphasizing the effect that antitrust enforcement has on cartels of firms with horizontally differentiated products.

In contrast to those, the novelty here is that we will combine an internal determinant of stability (product differentiation) with an exogenous determinant (the antitrust enforcement) in a Cournot-Bertrand model. The equilibrium will be evaluated by different probabilities of detection and by different degrees of product differentiation.

The initial point are the classical contributions of [Cournot \(1838\)](#) where firms choose a quantity to produce and [Bertrand \(1883\)](#) where the strategic variable chosen by firms is the price. Of course, that the law of supply and demand also remains in an oligopoly, a high level of output results in a relatively low price. Therefore, choose price or quantity refers only to the initial decision of firms.

Here we used a Cournot-Bertrand model formed by two firms that produce horizontally differentiated products. This model combines the Cournot and the Bertrand characteristics, as the same suggested. The firm 1 chooses quantity and firm 2 chooses price. This type of interaction was studied by [Tremblay and Tremblay \(2011\)](#) using the duopoly model incorporated by [Dixit \(1979\)](#), highlighting that the Nash equilibrium is stable when its products are considerably differentiated.

¹According to [Spagnolo \(2008\)](#), cartels are considered a mild form of organized crime, this illegal activity requires the joint effort of various agents to restrict competition by setting prices, allocated market shares, preventing entry, etc.

In contribution to the literature on economic collusion, we will investigate the following aspects. First, the behavior of these two firms to maintain the stability of the cartel – asking which one has higher impatience, as captured in the discount factor. Second, the effects of an exogenous disturbance, the antitrust enforcement, on the cartels' stability considering different degrees of product heterogeneity.

The rest of the paper is organized as follows. Section 2 presents the model describing each variable. Section 3 describes the behavior of the cartel members to maintain the agreement. Section 4 describes the impact of antitrust enforcement on Cournot-Bertrand cartels. Section 5 concludes and indicates some policy implications based on our results.

2 The Model

Consider two firms, 1 and 2, that produce horizontally differentiated products whose inverse demand functions are $p_1 = a - b(q_1 + \theta q_2)$ and $p_2 = a - b(q_2 + \theta q_1)$, with degree of product differentiation $\theta \in (0, 1)$, the products are homogeneous when $\theta = 1$, firms are monopolist when $\theta = 0$.

Assume that firm 1 chooses quantity, firm 2 chooses price. Reorganizing, we obtain the inverse demand function for firm 1 and the demand function for firm 2 respectively

$$p_1 = a(1 - \theta) - (1 - \theta^2)p_2 + \theta^2 q_1, \quad q_2 = a - p_2 - \theta q_1,$$

this firms operating in the market with marginal costs equal to c , they aim to maximize their profit functions defined as follows.

$$\pi_1 = (p_1 - c)q_1, \quad \pi_2 = (p_2 - c)q_2.$$

According to the behavior of the firms in the market, whether due to competition, cooperation, and diversion, they may capture different profits. Next, it is detailed, how the firms obtain profits according to their behavior:

- Firms competition leads to a strategic game, the solution of which is the Nash equilibrium, leaving firm 1 with a profit $\pi_1^N = \frac{(1 - \theta^2)(2 - \theta)^2(a - c)^2}{b(4 - 3\theta^2)^2}$ and firm 2 with a profit $\pi_2^N = \frac{((2 + \theta)(1 - \theta))^2(a - c)^2}{b(4 - 3\theta^2)^2}$;
- The cooperation or collusive agreement between firms leads to maximize their joint profits, each firm benefiting from a profit $\pi_1^C = \pi_2^C = \frac{(a - c)^2}{4b(1 + \theta)}$;
- Firms could also deviate from the collusive agreement, if firm 1 deviates from the agreement it will obtain a profit $\pi_1^D = \frac{(2 - \theta)^2(a - c)^2}{16b(1 + \theta^2)}$, harming firm 2 with a profit $\pi_{2D} = \frac{(2 - 2\theta - \theta^2)(a - c)^2}{8b(1 - \theta^2)}$, and if firm 2 deviates from the agreement, it will obtain a profit $\pi_2^D = \frac{(2 + \theta)^2(a - c)^2}{16b(1 + \theta)^2}$, leaving injured to firm 1 with a profit $\pi_{1D} = \frac{(2 + 2\theta - \theta^2)(a - c)^2}{8b(1 + \theta)^2}$.

The behavior of firms in the market leads us to a stage game G , called the cartel dilemma, with cooperate and non-cooperate actions induced by decision on quantities and price. From this game stage G , the infinitely repeated games will be generated that will be detailed in Sections 3 and 4.

3 Stability Cartel

In this section, we will study the stability of the Cournot-Bertrand cartel on different degrees of product differentiation considering that the Cournot-Bertrand cartel operates in the market knowing that there is no antitrust authority that can investigate, prosecute, fine or convict it for its illegal activity.

Given collusive agreements or cartels are illegal activities and cannot have a legal agreement between its members. The decision to keep in a cartel of firms can be represented by a repeated game, [Friedman \(1971\)](#) inserts a rule for maintaining collusive agreement for two firms $i, j \in \{1, 2\}$ called a trigger strategy defined as follows:

- If firm i cooperates in the previous period, firm j cooperates in the period present.
- If firm i does not cooperate in the previous period, firm j competes in the present period and forever.

This rule and notion of perfect sub-game equilibrium leads us to the following incentive compatibility constraint.

$$V^C \geq V^D \iff \delta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N},$$

where V^C is the liquid present value maintain cartel, V^D the liquid present value of deviating in the first stage and competing in the later stages and δ the discount factor, such that $0 < \delta < 1$. The most impatient firms are those with the highest discount factor, i.e. firms that value present revenues more than future ones. Therefore the degree of indebtedness can be a determinant for the stability of the cartels by making the firms need to value the present revenues more.

From the game stage or cartel dilemma of Section 2, for each firm we have the following incentive compatibility constraints.

$$\delta_1(\theta) \geq \frac{(4 - 3\theta^2)^2}{(8 - 7\theta^2)(2 - \theta)^2}, \quad \delta_2(\theta) \geq \frac{(4 - 3\theta^2)^2}{(8 - 7\theta^2)(2 + \theta)^2}. \quad (1)$$

Considering that for each degree of horizontal product differentiation $\theta \in (0, 1)$ there are Cournot-Bertrand collusive agreements. The inequalities in (1) indicate that these collusive agreements remain stable at different levels of the discount factor.

Proposition 3.1. *Given the Cournot-Bertrand collusive agreements. Cournot-type firms are more impatient than Bertrand-type firms, regardless of the degree of product differentiation.*

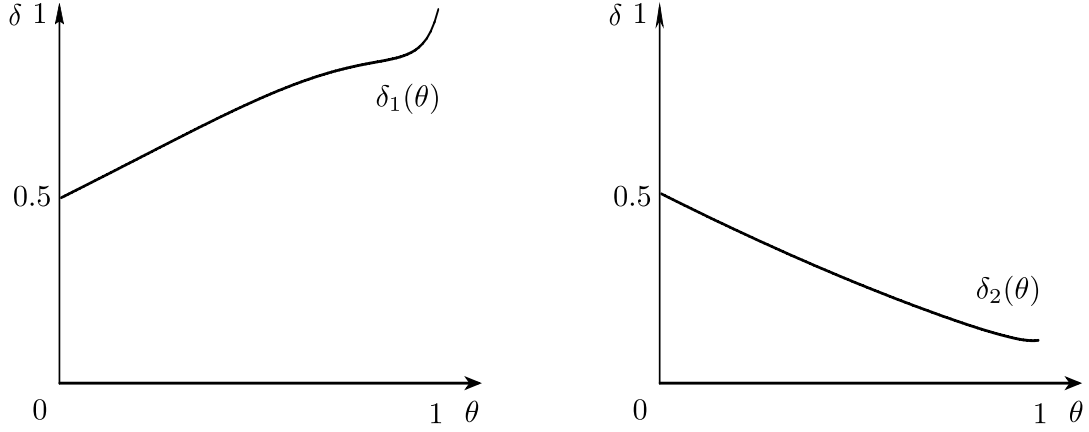


Figure 1: Firms behavior to maintain cartel stability.

Proof. To prove it is enough to verify that $\delta_1(\theta) > \delta_2(\theta)$, for all $\theta \in (0, 1)$. Indeed: for all $\theta \in (0, 1)$ satisfies

$$\begin{aligned} (2 + \theta)^2 > (2 - \theta)^2 &\implies \frac{1}{(2 - \theta)^2} > \frac{1}{(2 + \theta)^2} \\ &\implies \frac{w}{(2 - \theta)^2} > \frac{w}{(2 + \theta)^2} \end{aligned}$$

where $w = \frac{(4 - 3\theta^2)^2}{(8 - 7\theta^2)}$. Therefore, $\delta_1(\theta) > \delta_2(\theta)$. \square

Proposition 3.2. *Given the Cournot-Bertrand collusive agreements, Cournot-type firms are more impatient when products are more homogeneous and Bertrand-type firms are less impatient when products are more homogeneous.*

Proof. Since the functions $\delta_1, \delta_2 : (0, 1) \rightarrow (0, 1)$ are differentiable. To prove it is sufficient verify that the functions δ_1 is increasing and δ_2 is decreasing in $(0, 1)$. Indeed: for all $\theta \in (0, 1)$ satisfies,

$$\delta_1'(\theta) = \frac{4\bar{w}(4 - 3\theta^2)}{(8 - 7\theta^2)^2(2 - \theta)^3} > 0, \quad \delta_2'(\theta) = \frac{4\bar{w}(4 - 3\theta^2)}{(8 - 7\theta^2)^2(2 + \theta)^3} < 0,$$

where $\bar{w} = 21\theta^3 + 16\theta^2 - 20\theta - 16$. \square

Figure 1 illustrates Propositions 2.1 and 2.2. It is observed that the Cournot-type firms operate at a higher discount factor level than the Bertrand-type firms and are more likely to destabilize the cartel when the products are more homogeneous.

4 Enforcement Antitrust

In this section, we will study the stability of the Cournot-Bertrand cartel on different degrees of product differentiation and different intensities of antitrust enforcement considering that the Cournot-Bertrand cartel operates in the market knowing that there is an antitrust authority that investigates and processes legally so that those responsible are fined and convicted of their illegal activity.

To insert enforcement antitrust into the dynamics of the cartel members we need to modify the grim-trigger strategy. From the game stage or cartel dilemma generated in the Section 2 and based on [Houba et al. \(2015\)](#) and [Harrington Jr \(2017\)](#), the modified grim-trigger strategy we described below.

Let V^C be the expected present value of the firms for maintaining the collusive agreement defined by the recursive dynamics:

$$V^C = \pi^C + \beta \left[-f\pi^C + \delta \frac{\pi^N(p^N)}{1-\delta} \right] + (1-\beta)\delta V^C,$$

where β is the probability of cartel detection, f the accumulated fine rate of each firm for acting as a cartel and δ the discount factor. On the other hand, let V^D be the liquid present value when the firm deviates in the first stage defined as:

$$V^D = \pi^D + \frac{\delta\pi^N}{1-\delta} = \frac{(1-\delta)\pi^D + \delta\pi^N}{1-\delta},$$

the perfect sub-game equilibrium and two present values lead us to the following incentive compatibility constraint for both firms:

$$V^C \geq V^D \iff \delta \geq \frac{\pi^D - (1-\beta f)\pi^C}{(1-\beta)[\pi^D - \pi^N]}.$$

Suppose that β represents enforcement antitrust such that $\beta \in (0, \bar{\beta}]$, with $\bar{\beta} < 1$ and cumulative fine rate for each firm is $\bar{f} = 0.2^2$. From the game stage or cartel dilemma of Section 2 and modified grim-trigger strategy, for each firm we have the following incentive compatibility constraints:

$$\bar{\delta}_1(\beta, \theta) \geq \frac{(4-3\theta^2)^2(\theta^2+2\beta-2\theta\beta)}{5\theta^2(1-\beta)(8-7\theta^2)(2-\theta)^2}, \quad \bar{\delta}_2(\beta, \theta) \geq \frac{(4-3\theta^2)^2(5\theta^2+4\beta+4\theta\beta)}{5\theta^2(1-\beta)(8-7\theta^2)(2+\theta)^2}. \quad (2)$$

The inequalities in (2) indicate that the sustainability of the collusive agreement depends on the degree of product differentiation and the intensity of the antitrust enforcement. Below is a result that characterizes the intensity of the impact of antitrust enforcement on cartels with horizontal product differentiation.

Proposition 4.1. *Given the presence of an enforcement antitrust, it impacts strongly on Cournot-Bertrand cartels with highly differentiated horizontal products.*

Proof. Since the functions $\bar{\delta}_1, \bar{\delta}_2 : (0, 1) \times [0, \bar{\beta}] \rightarrow (0, 1)$ are differentiable. To prove it is sufficient verify that the functions $\bar{\delta}_1$ and $\bar{\delta}_2$ are increasing in $[0, \bar{\beta}]$ for all $\theta \in (0, 1)$. Indeed: for each $\beta \in [0, \bar{\beta}]$ satisfies

$$\bar{\delta}'_1(\beta, \theta) = \frac{\delta_1(\theta)(\theta^2 - 2\theta + 2)}{\theta^2(1-\beta)^2} > 0, \quad \bar{\delta}'_2(\beta, \theta) = \frac{\delta_2(\theta)(5\theta^2 + 4\theta + 5)}{5\theta^2(1-\beta)^2} > 0,$$

for all $\theta \in (0, 1)$. □

²The reference value follows the fine for collusion in Brazil. For more information, see Brazilian Antitrust Agency (CADE) - Artigo 37 da Lei n° 12.529 de 30 de Novembro de 2011.

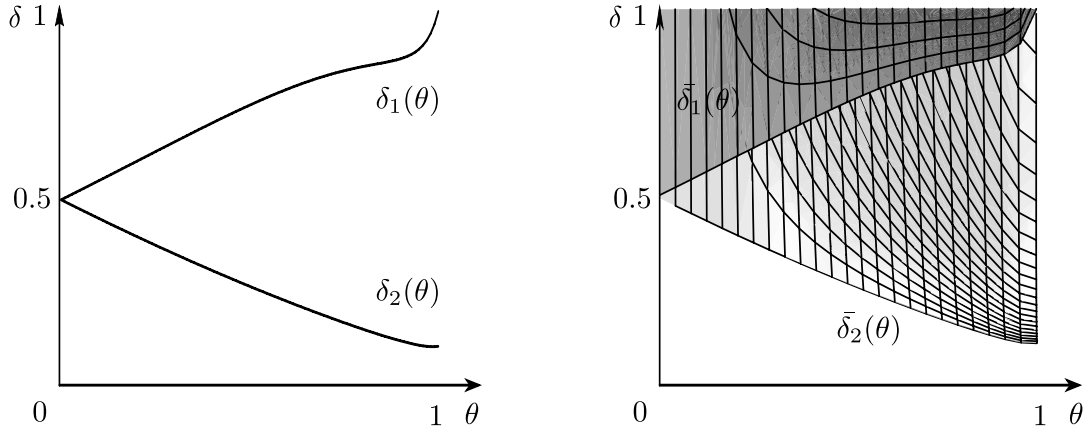


Figure 2: Impact of antitrust enforcement on cartels stability.

Figure 2 illustrates Proposition 4.1. It is observed that given the antitrust enforcement, the Cournot-type and Bertrand-type firms easily destabilize the agreement when their products are highly differentiated, that is, the discount factor of the Cournot-type firm is quadratic and convex in $(0, 1)$, for all $\beta \in (0, \bar{\beta}]$ and the discount factor of the Bertrand-type firms is decreasing in $(0, 1)$, for all $\beta \in (0, \bar{\beta}]$. Therefore, given the antitrust enforcement, the Cournot-Bertrand cartels are more susceptible to destabilization when their products are highly differentiated.

5 Conclusion

In this work, we observe how the degree of differentiation of products impacts on the stability of a Cournot-Bertrand cartel. In addition, the cartel's stability was also evaluated at different intensities of antitrust efforts.

In the absence of enforcement antitrust, we showed that cartels remain stable when the Cournot-type firm has an increasing discount factor and the Bertrand-type firm has a decreasing discount factor. We also demonstrate that antitrust enforcement impacts more Cournot-Bertrand type cartels when their horizontal products are highly differentiated.

As a recommendation to guide the activity of the antitrust authority is to intensify the investigations on possible collusive agreements in markets where products are more homogeneous.

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