Cartel Stability Under Antitrust Enforcement and Product Quality Differentiation*

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Resumo

O conluio ocorre em diferentes mercados com diferentes níveis de diferenciação da qualidade do produto. Este artigo examina o trade-off entre o grau de diferenciação da qualidade do produto e a sustentabilidade do cartel — um cartel formado por uma empresa que produz produtos de alta qualidade e outra de baixa qualidade com uma estratégia de conluio de fixação de preços —, e o efeito do antitrust enforcement sobre a estabilidade do cartel em diferentes níveis de diferenciação da qualidade do produto. Mostramos que a empresa que produz produtos de baixa qualidade tem mais probabilidade de desestabilizar o cartel do que a empresa que produz produtos de alta qualidade. Além disso, mostramos que quando os produtos são altamente diferenciados e tendem a ser homogêneos, o cartel fica menos estável. Em seguida, provamos que o antitrust enforcement afeta negativa e uniformemente sob estabilidade do cartel, ou seja, não afeta fortemente a nenhum determinado grau de diferenciação da qualidade do produto.

Palavras-chave: Product Quality Differentiation · Bertrand Model · Trigger Strategy · Antitrust Enforcement · Sustainability Cartel.

Abstract

Collusion occurs in different markets with different levels of product quality differentiation. This paper examines the trade-off between product quality differentiation degree and the cartel sustainability — a cartel formed by a firm that produces high-quality products and another of low-quality with a collusive price-setting strategy —, and the effect of antitrust enforcement on the stability of the cartel at different levels of product quality differentiation. We show that the firm that produces low-quality products is more likely to destabilize the cartel than the firm that produces high-quality products. Besides, we show that when products are highly differentiated and tend to be homogeneous, the cartel is less stable. Then, we prove that enforcement affects negatively and uniformly on the stability of the cartel, that is, it does not strongly affect a specific degree of product quality differentiation.

Keywords: Product Quality Differentiation · Bertrand Model · Trigger Strategy · Antitrust Enforcement · Sustainability Cartel.

JEL: L13 · L41 · D43 · C73.

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1 Introduction

In economic theory, collusion is when the firms of an industry agree on a behavior that leads them to obtain supra-competitive profits — profits above competitive profits. In collusive behavior, the firms use strategies that depend on a history to maintain supra-competitive results through a reward-punishment scheme, which rewards the firm that maintains the supra-competitive result and punishes the firm that deviates from the agreement (Harrington Jr, 2017). In this sense, many study the stability of cartels affected by factors intrinsic to the market structure such as cost differences (Rothschild, 1999; Bos and Marini, 2019), degree of product differentiation (Häckner, 1994; Andaluz, 2010; Bos and Marini, 2019), number of participants in the cartel. On the other hand, from the point of view of the defense of competition, cartels are considered a type of organized crime that damages the economic order, mainly harms consumers with high prices. The economies, to combat anti-competitive activities approve their competition or antitrust law to provide greater social welfare. Thus, show that antitrust enforcement negatively affects the stability of cartels (Block et al., 1981; Houba et al., 2015).

Advancing in literature, this paper aims to analyze the trade-off between the degree of product quality differentiation and the stability of the cartel — a cartel with a collusive price-setting strategy—, and the effect of antitrust enforcement on the stability of the cartel at different levels of product quality differentiation. Analyzing this topic has the utmost importance because the cartels can be constituted in different industries and with differentiated products. Therefore, this analysis helps guide the activity of the antitrust authority to focus its investigation on more stable cartels.

To develop the trade-off between the degree of product quality differentiation and the sustainability of the cartel — a cartel made up of a firm that produces low-quality products and the other firm that produces high-quality products with a collusive price-setting strategy —, and the effect of antitrust enforcement on the stability of the cartel at different levels of product quality differentiation we used the repeated games theory, more precisely, the grim-trigger strategy and modified grim-trigger strategy respectively.

The trade-off between the degree of product quality differentiation and the sustainability of the cartel is constantly explored in literature. Häckner (1994),Bos and Marini (2019), show that there is no clear behavior between product quality differentiation degree and the sustainability of the cartel, which means, there is no clear behavior of the discount factor in the product quality differentiation interval. A priory we show that the firm that produces high-quality products is less susceptible to destabilizing the cartel than the firm that produces low-quality products, which implies that the sustainability of the cartel depends more on the firm that produces low-quality product than on the one that produces produces high-quality products. Consequently, we show that the cartel is sustainable in every degree product quality differentiation, but the stability of the cartel is linked to the degree of product quality differentiation — improves or worsens the stability of the cartel. Unlike Häckner (1994),Andaluz (2010) we show that when the products are highly differentiated and tend to be homogeneous, the cartels are less stable, and when they are moderately differentiated, the cartel are more stable.

The effect of antitrust enforcement on the stability of cartels is widely explored in literature. Block et al. (1981), Motta and Polo (2003), and Houba et al. (2015) show that antitrust enforcement negatively affects the stability of cartels. If the cartel has a collusive price-setting strategy, antitrust enforcement causes the collusive price to decrease, only in this way will the cartel remain sustainable. As far as we know, no one explores the effect of antitrust enforcement on the stability of the cartel at different levels of product quality differentiation. We show, that for each degree of product quality differentiation, antitrust enforcement negatively affects the stability of the cartel uniformly. Therefore, the first affected would be the cartels with products that tend to be homogeneous.

The rest of the paper is organized as follows. Section 2 describes a demand system with vertically differentiated products. Section 3 describes the profit capture of firms based on their behavior. Section 4 describes the trade-off between product quality differentiation degree and the sustainability of the cartel. Section 5 describes the effect of antitrust enforcement on the stability of the cartel at different levels of product quality differentiation. Section 6 describes the numerical simulation of the effect of antitrust enforcement on the stability of the cartel at different levels of the fine rate. Finally, Section 7 concludes and includes recommendations to guide antitrust authority activity.

2 Model

Consider a market with vertically differentiated products — products differentiated by quality — served by a duopoly, firms 1 and 2, such that firm 2 produces high-quality products and firm 1 produces low-quality products, and the product differentiation parameters have the following relation respectively: $s_2 \geq s_1$. The valuation of the products and the distribution structure of consumers consists of: (i) the consumers value the products with the parameter $\theta \in [a,b] \subset [1,\infty)$; (ii) consumers are evenly distributed in $[a,b]$ with mass normalized to one; (iii) consumers hardly buy a product. The consumer located in $\theta \in [a,b]$ has the utility function defined as:

$$U(\theta) = \theta s_i - p_i, \tag{1}$$

where $p_i \in [0, s_i b]$ are the prices of the products $i \in \{1,2\}$. The existence of the indifferent consumer between consuming product 1 or product 2 leads to obtaining the same level of utility, that is:

$$\theta s_1 - p_1 = \theta s_2 - p_2 \Longrightarrow \theta = \frac{p_2 - p_1}{s_2 - s_1}. \tag{2}$$

Since any consumer located in $\theta \in [a,b]$ can be indifferent between consuming the product 1 and product 2, it generates a demand system that is served by firms 1 and 2, that is:

$$D_1(p_1,p_2) = \theta - a = \frac{p_2 - p_1}{s_2 - s_1} - a, \qquad D_2(p_1,p_2) = b - \theta = b - \frac{p_2 - p_1}{s_2 - s_1}. \tag{3}$$

Expressing in this way the demand system 3 that characterizes a vertically differentiated product market structure is conducive to Bertrand competition.

3 Firms Behavior

The behavior of firms in the market leads to obtaining different profits in the market[1], the profit of a cooperative behavior exceeds the profit of competitive behavior; instead, the profit from deviating from a cooperative behavior exceeds the profit from cooperation, left to the firm that cooperates with a profit less than the competitive profit. The strategic interaction of the Bertrand type of the vertically differentiated duopoly consists of:

- Firms: 1 and 2;
- Prices choice: $p_i \in [0, p_{max}]$, for all $i \in \{1,2\}$;
- Payoff functions: $\pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$, for all $i, j \in \{1,2\}$ with $i \neq j$.

where $c = 0$ is the marginal cost with which firms operate in the market, $D_i(p_i, p_j)$ is the demand system 3 for all $i, j \in \{1,2\}$ with $i \neq j$. According to the firms' behavior in the market, whether due to competition, cooperation, and diversion, they may capture different profits. Next, the way that the firms obtain profits according to their behavior is presented.

Lemma 3.1. *Consider that firms 1 and 2 serve demand system 3. The Nash-Bertrand equilibrium profile is:*

$$p_1^N = \frac{2a(s_1 - s_2) + 2b(s_2 - s_1)}{3}, \quad p_2^N = \frac{a(s_1 - s_2) + 2b(s_2 - s_1)}{3}, \tag{4}$$

generating competitive profits for firms 1 and 2 respectively.

$$\pi_1^N = \frac{(s_2 - s_1)(b - 2a)^2}{9}, \quad \pi_2^N = \frac{(s_2 - s_1)(a - 2b)^2}{9}. \tag{5}$$

Proof. The solution for a competitive environment is the individual profit maximization of the firms. Then the situation is solved as a simultaneous game and the Nash equilibrium can be written as the system of best responses:

$$p_1 = \frac{p_2 + a(s_1 - s_2)}{2}, \quad p_2 = \frac{p_1 + b(s_2 - s_1)}{2}. \tag{6}$$

Then, solving the system of best responses, the Bertrand-Nash equilibrium profile (p_1^N, p_2^N) is obtained. Consequently, evaluating (p_1^N, p_2^N) in the profit functions, the competitive profits are π_1^N and π_2^N. □

The following result is due to a cooperative behavior, where the maximum collective profit of the firms and the joint profit of the industry are found.

Lemma 3.2. *Consider that firms 1 and 2 serve demand system 3. The optimal cartel price vector is:*

$$p_1^C = as_1, \quad p_2^C = as_1 + \frac{b}{2}(s_2 - s_1), \tag{7}$$

generating cooperative profits for firms 1 and 2 respectively.

$$\pi_1^C = \frac{as_1(b - 2a)}{2}, \quad \pi_2^C = \frac{b(bs_2 - bs_1 + 2as_1)}{4}. \tag{8}$$

[1] All this mechanism leads to a cartel dilemma game with cooperate and non-cooperate actions for both firms.

Proof. To capture the prices that induce supra-competitive profits — maximization of joint profits. Häckner (1994) sets the price of firm 1 with value $p_1^C = as_1$. Then, maximizing the profit function $\pi = \pi_1(p_1^C, p_2) + \pi_2(p_1^C, p_2)$ with respect to p_2, by first-order conditions we have:

$$\frac{2p_2 - 2as_1 + bs_1 - bs_2}{s_1 - s_2} = 0. \tag{9}$$

Solving equation 9, we obtain the cartel price p_2^C of firm 2. Then, evaluating the vector of cartel prices in the profit functions, we obtain those of cartel prices for both firms π_1^C and π_2^C respectively. □

The following two results explain that the deviation of some firms, given that, the others maintain the collusive strategy, obtains greater profits than the cooperative profit and leaves those who maintain the collusive strategy with less profit than competitive profit.

Lemma 3.3. *Consider that firms 1 and 2 serve demand system 3. Since firm 2 cooperates, firm 1 optimal deviation price is:*

$$p_1^D = \frac{2s_1 - s_2}{2} + \frac{s_2 - s_1}{4}, \tag{10}$$

generating profit from diversion of firm 1 and profit from damage of firm 2 respectively.

$$[0.97]\pi_1^D = \frac{(2a(s_1 - s_2) + b(s_2 - s_1))^2}{16(s_2 - s_1)}, \quad \pi_{2D} = \frac{(2as_2 - 3b(s_2 - s_1))(2as_1 + b(s_2 - s_1))}{8(s_2 - s_1)}. \tag{11}$$

Proof. To find the deviation price of firm 1, set the cartel price of firm 2. By first-order conditions of the function $\pi_1(p_1, p_2^C)$ with respect to p_1, we obtain:

$$\frac{4p_1 - 4as_1 + 2as_2 + bs_1 - bs_2}{2(s_1 - s_2)} = 0 \tag{12}$$

Then, solving equation 3, we obtain the optimal deviation price p_1^D — since $\pi_1(p_1, p_2^C)$ is concave, then the critical point is optimal. Therefore, evaluating the optimal price p_1^D in $\pi_1(p_1, p_2^2)$, we obtain the profit from deviation π_1^D of firm 1 and the profit of damage π_{2D} of firm 2. □

Lemma 3.4. *Consider that firms 1 and 2 serve demand system 3. Since firm 1 cooperates, firm 2 optimal deviation price is:*

$$p_2^D = \frac{b(s_2 - s_1) + as_1}{2}, \tag{13}$$

generating profit from diversion for firm 2 and profit from damage for firm 1 respectively.

$$\pi_2^D = \frac{(as_1 + b(s_2 - s_1))^2}{4(s_2 - s_1)}, \quad \pi_{1D} = \frac{as_1(b(s_2 - s_1) - a(s_2 - s_1))}{2(s_2 - s_1)}. \tag{14}$$

Proof. To find the deviation price of firm 2, set the cartel price of firm 1. By first-order conditions of the function $\pi_2(p_1^C, p_2)$ with respect to p_2, we obtain:

$$\frac{2p_2 - as_1 + bs_1 - bs_2}{s_1 - s_2} = 0. \tag{15}$$

Then, solving equation k, we obtain the optimal deviation price p_2^D — since $\pi_2(p_1^C, p_2)$ is concave. Then the critical point is optimal. Therefore, evaluating the optimal price p_2^D in $\pi_2(p_1^C, p_2)$, the result is the profit from deviation π_2^D of firm 2 and the profit of damage π_{1D} of firm 1. □

The capture of the profits found in Lemmas 3.1, 3.2, 3.3, and 3.4, generate the dilemma of the cartel[2] — normal game or stage game with cooperate (C) and non-cooperate (NC) actions for firms 1 and 2. This stage game generates infinitely repeated game, in particular, the sub-game called grim-trigger strategy.

4 Cartel Stability

Here, we study the trade-off between product differentiation degree and the sustainability or stability of the cartel based on the perfect equilibrium of the sub-game of the grim-trigger strategy developed by Friedman (1971), this being a sub-game of the infinitely repeated game generated by the cartel dilemma.

From the perfect sub-game equilibrium of the grim-trigger strategy, this being a sub-game of the infinitely repeated game generated by the cartel dilemma, we conceptualized the notion of cartel stability with N participants. A cartel made up of N asymmetric firms is said to be stable if for all $i \in N$ there exists $\delta \in (0,1)$ such that $\delta = \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\}$ and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \iff \delta \geq \delta_i \geq \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N}, \tag{16}$$

where V_i^C is the liquid present value which maintains the cartel, V_i^D is the liquid present value of deviating in the first stage and competing in the later stages and $\delta_i \in (0,1)$ is the discount factor. The minimum discount factor $\delta_i \in (0,1)$ with which firm $i \in N$ maintains the stability of the cartel is called the critical discount factor[3] of firm $i \in N$. The set of discount factors with which firm $i \in N$ maintains the stability of the cartel defined as $\{\delta \in (0,1) : \delta_i \leq \delta < 1\}$ is called the sustainability interval of firm $i \in N$. Taking as a reference the critical discount factor with which the firms maintain the stability of the cartel, the

[2] The bimatricial representation of the cartel dilemma is:

$$\begin{pmatrix} \pi_1^C, \pi_2^C & \pi_{D1}, \pi_2^D \\ \pi_1^D, \pi_{D2} & \pi_1^N, \pi_2^N \end{pmatrix}$$

where the first row (column) are the payoffs of the cooperate action of firm 1 (firm 2) and the second row (column) are the payoffs of the non-cooperate action of firm 1 (firm 2).

[3] The most impatient firms are those with the highest discount factor, i.e., firms that present value revenues more than future ones. Therefore, the degree of indebtedness can be a determinant for the cartels' stability by making the firms need to value the present revenues more.

following definition captures which of the participating firms of the cartel is more sensitive to destabilizing the cartel[4].

Definition 4.1. Consider a cartel made up of N asymmetric firms. Firm $j \in N$ is more susceptible to destabilizing the cartel if for all $i \in N - \{j\}$ with $i \neq j$, $\delta_i < \delta_j$, i.e., firm j maintains the stability of the cartel at a higher critical discount factor than firms $i \in N$.

To characterize the incentive compatibility constraint of a cartel made up of a duopoly, firms 1 and 2, which serve the demand system 3, we make the following assumption.

Assumption 4.2. Consider a cartel made up of a duopoly, firms 1 and 2, which serve demand system 3 for all degrees of product quality differentiation $\theta = \frac{s_1}{s_2} \in (0,1)$, knowing that there is no antitrust enforcement.

Found the profits (payoff) of the cartel's dilemma in Section 2, we evaluate the profits in the incentive compatibility constraint (ICC) 17. The cartel made up of firms 1 and 2 is stable if exists $\delta \in (0,1)$ such that $\delta = \max\{\delta_1, \delta_2\}$ and satisfies the following incentive compatibility constraint (ICC)[5]:

$$\delta_1(\theta) \geq \frac{9(2.1\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007}, \quad \delta_2(\theta) \geq \frac{9\theta^2}{-27.301\theta^2 + 36.421\theta - 0.1207}, \tag{17}$$

where the inequality on the left side is the ICC of the firm that produces low-quality products (firm 1), and the inequality on the right side is the ICC of the firm that produces high-quality products (firm 2).

To characterize the stability of the cartel at different levels of product quality differentiation, a priory we diagnosed which of the participating cartel firms is more susceptible to destabilizing the cartel. By Definition 4.1, the most sensitive firm to destabilizing the cartel is the firm that maintains the stability of the cartel at a high critical discount factor.

Proposition 4.3. *Consider that for every degree of product quality differentiation $\theta \in (0,1)$ of demand system 3, firms 1 and 2 cartilize the market with a collusive price-setting strategy. The firm that produces low-quality products (firm 1) is more likely to destabilize the cartel than the firm that produces high-quality products (firm 2).*

Proof. To show this conjecture, it is enough to prove that for every degree of product quality differentiation $\theta \in (0,1)$, the critical discount factor of firm 1 is greater than that of firm 2. Indeed: since $9(2.1\theta - 0.01)^2 > 9\theta^2$, and $-27.301\theta^2 + 36.421\theta - 0.1207 > 35.639\theta^2 + 0.3614\theta - 0.0007$, for all $\theta \in (0,1)$,

$$\frac{9(2.1\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007} > \frac{9\theta^2}{-27.301\theta^2 + 36.421\theta - 0.1207}.$$

Therefore, $\delta_1(\theta) > \delta_2(\theta)$, for all $\theta \in (0,1)$. □

[4] See Bruttel (2009).

[5] Without loss of generality consider the following parameters in the market structure $a = 1$, $b = 2.01$, and $\theta = \frac{s_1}{s_2}$. If θ is close to "1" the products tend to be homogeneous; if θ is close to "0", the vertical products are highly differentiated.

This result shows that the sustainability of the cartel depends more on the firm that produces low-quality products (firm 1) than on the firm that produces high-quality products (firm 2).

Proposition 4.3 diagnoses the firm most sensitive to destabilizing the cartel. The following result characterizes the trade-off between product quality differentiation degree and the sustainability of the cartel, we analyze the trade-off between the critical discount factor of firm 1 and product quality differentiation degree, that is, the behavior of the critical discount factor function of firm 1 defined in the product quality differentiation interval[6].

Proposition 4.4. *Consider that for every degree of product quality differentiation* $\theta \in (0,1)$ *of demand system 3, firms 1 and 2 cartilize the market with a collusive price-setting strategy. The cartel is less stable when the products are almost homogeneous and highly differentiated, i.e., the critical discount factor function is quadratic and reaches its minimum over the interior of the product differentiation interval.*

Proof. To prove this proposition, it is sufficient to show that the critical discount factor function $\delta_1 : (0,1) \longrightarrow (0,1)$ defined as:

$$\delta_1(\theta) = \frac{9(2.01\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007},$$

it is quadratic. Indeed: By first-order conditions we have:

$$\frac{1.3018 \times 10^9\theta^2 - 2.866 \times 10^7\theta + 1.1037 \times 10^5}{M} = 0,$$

where $M = 6.3507 \times 10^{10}\theta^4 + 1.2880 \times 10^9\theta^3 - 1.8417 \times 10^7\theta^2 - 2.5298 \times 10^5\theta + 2450.0$. Solving the equation 18 we obtain the root $\bar{\theta} = 0.01704$. Then substituting $\bar{\theta} = 0.01704$ in $\delta_1(\theta)$ we obtain 0.55675. Therefore, δ_1 reaches a minimum in the product differentiation interval $(0,1)$. □

The trade-off between product quality differentiation degree — vertically differentiated products — and the suitability of the cartel — cartel with a price-setting collusive strategy — is widely explored in literature by Häckner (1994), Andaluz (2010). In general, they do not reach a consensus on the trade-off between the degree of product differentiation and the sustainability of the cartel, which means, there is no clear behavior of the critical discount factor function on the product differentiation interval. Intuitively, the correct thing is that when quality products are highly differentiated, cartels are less sustainable.

Figure 1 illustrates Propositions 4.3 and 4.4. On one hand, it is observed that for each degree of product quality differentiation, the critical discount factor of firm 1 is greater than the critical discount factor of firm 2. Similarly, the critical discount factor function of firm 1 is greater than that of firm 2 over the product differentiation interval. In other words, firm 1 is more sensitive to destabilizing the cartel because it has more incentives to deviate from the cartel than firm 2. On the other hand, it is noted that the sustainability of the cartel depends more on the firm that produces low-quality products (firm 1) than the firm that produces high-quality products (firm 2). Finally, note Figure 1 and the regions — regions where the

[6] The product quality differentiation interval is the set $\{\theta : 0 < \theta < 1\}$, such that, if θ is close to 1, the products are almost homogeneous; and if θ is close to 0, the products are highly differentiated.

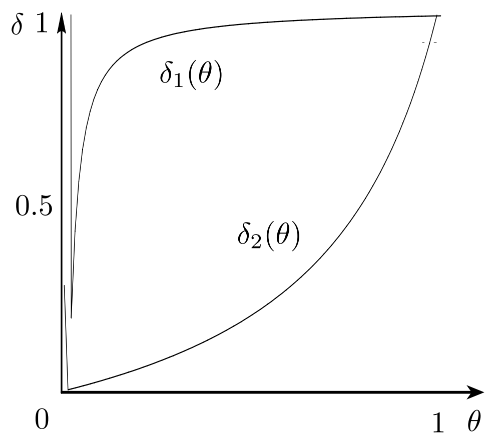

Figure 1: Trade-off between the degree of product quality differentiation and the sustainability of the cartel made up of firm 1 and 2 with a collusive price-setting strategy.

levels of discount factors that support the stability of the cartel are found — defined as:

$$1-\int_0^1 \frac{9(2.1\theta-0.01)^2}{35.639\theta^2+0.3614\theta-0.0007}d\theta,\ 1-\int_0^1 \frac{9\theta^2}{-27.301\theta^2+36.421\theta-0.1207}d\theta, \tag{18}$$

that for each degree of product quality differentiation $\theta \in (0,1)$, the sustainability interval of firms 1 and 2 has the following relationship: $[\delta_1(\theta),1) \subset [\delta_2(\theta),1)$. This means that the sustainability interval of firm 1 is less than that of firm 2, more precisely, firm 1 is more susceptible to destabilizing the cartel and that the sustainability of the cartel depends more on the behavior of firm 1 than that of firm 2.

5 Cartel Stability Under Antitrust Enforcement

This section studies the stability of a cartel with a collusive price-setting strategy under antitrust enforcement — represented by the probability of detection of the cartel and the fine rate for participating in the cartel — at different levels of product quality differentiation.

To model this scenario we use the infinitely repeated game theory, more precisely, the modified grim-trigger strategy, this being a sub-game of the infinitely repeated game generated by the cartel dilemma. Next, we mention the modified grim-trigger strategy for a cartel context with $N=\{1,2,\cdots,n\}$ participating firms (Motta and Polo, 2003; Houba et al., 2015). Denote as V_i^C the expected present value of firm $i \in N$ when maintaining the collusive agreement, and define as the following recursive dynamic:

$$V_i^C = \pi_i^C + \beta\left[-f\pi_i^C + \bar{\delta}_i \frac{\pi_i^N}{1-\bar{\delta}_i}\right] + (1-\beta)\bar{\delta}_i V_i^C, \tag{19}$$

where β represents the probability of detection of the cartel, f represents the fine rate of firm $i \in N$, and $\bar{\delta}$ is the inter-temporal discount factor. On the other hand, denote as V_i^D the liquid present value of the deviation of firm i when deviating from the collusive agreement or cartel, and define as:

$$V_i^D = \pi_i^D + \frac{\bar{\delta}_i \pi_i^N}{1-\bar{\delta}_i} = \frac{(1-\delta)\pi_i^D + \bar{\delta}_i \pi_i^N}{1-\bar{\delta}_i}. \tag{20}$$

Since $1-\bar{\delta}_i > 0$, $1-\beta(1-\bar{\delta}_i) > 0$, and the perfect sub-game equilibrium of the modified grim-trigger strategy, cartel stability under antitrust enforcement is conceptualized. A cartel is said to be stable under antitrust enforcement if, for each firm i there exists $\bar{\delta} \in (0,1)$, such that $\bar{\delta} = \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\}$, and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \Longleftrightarrow \bar{\delta}_i \geq \frac{\pi_i^D - (1-\beta f)\pi_i^C}{(1-\beta)[\pi_i^D - \pi_i^N]}. \tag{21}$$

The minimum discount factor $\bar{\delta}_i$ with which the firm $i \in N$ maintains the stability of the cartel under antitrust enforcement is called the critical discount factor under antitrust enforcement — discount factor satisfying equation 21. The set of discount factors with which firm $i \in N$ maintains the stability of the cartel under antitrust enforcement, defined as $\{\delta \in (0,1) : \bar{\delta}_i \leq \delta < 1\}$, is called the sustainability interval under antitrust enforcement of the firm $i \in N$.

To characterize the effect of antitrust enforcement on the stability of cartels, the critical discount factor that sustains the cartel in a free field of antitrust enforcement is taken as a basis. See the definition below.

Definition 5.1. Consider a cartel made up of N asymmetric firms. Antitrust enforcement negatively affects the stability of the cartel if the maximum critical discount factor of firms $i \in N$ $\delta = \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\}$ is less than the maximum critical discount factor of firm $i \in N$ under antitrust enforcement, i.e., $\delta < \bar{\delta}$.

The following characterizes the effect of antitrust enforcement on cartel stability, with a collusive price-setting strategy at different product quality differentiation levels.

Assumption 5.2. Consider a cartel made up of a duopoly, firms 1 and 2, which serve demand system 3 for all degrees of product quality differentiation $\theta = \frac{s_1}{s_2} \in (0,1)$, knowing that there is an antitrust enforcement with parameter $\bar{f} = \min\{f : 0.01 \leq f \leq 0.20\}$.

The cartel made up of firms 1 and 2 is stable under antitrust enforcement, if for each firm $i \in \{1,2\}$ there is $\bar{\delta} \in (0,1)$, such that $\bar{\delta} = \max\{\bar{\delta}_1, \bar{\delta}_2\}$ and satisfies the incentive compatibility constraints:

$$\bar{\delta}_1(\theta) = \frac{(3.6361 \times 10^5\theta^2 - 36180\theta + 9) - 72\beta\theta(1-\theta)}{(3.5639 \times 10^5\theta^2 + 36140\theta - 7) - \beta(3.5639 \times 10^5\theta^2 + 36140\theta + 7)}, \tag{22}$$

where $\bar{\delta}_1(\theta)$ is the incentive compatibility constraint of the firm that produces low-quality products and $\bar{\delta}_2(\theta)$ defined as:

$$\frac{(904500\theta^2 - 1.8181 \times 10^7\theta + 90450) - \beta(1809\theta^2 - 3.6542 \times 10^5\theta + 3.6361 \times 10^5)}{(3.621 \times 10^7\theta^2 - 1.8241 \times 10^5\beta + 30250.0) + \beta(3.621 \times 10^7\theta^2 - 1.8241 \times 10^7\theta + 30250)}, \tag{23}$$

is the incentive compatibility constraint of the firm that produces high-quality products.

The following result explains the effect of antitrust enforcement on the stability of the cartel— cartel with collusive price-setting strategy— at different levels of product quality differentiation. Proposition 4.3 diagnoses which of the firms is the most susceptible to destabilizing the cartel, which implies that the effect of antitrust enforcement on the stability of the cartel has to be analyzed on the incentives compatibility constraint of the firm that produces low-quality products.

Proposition 5.3. *Consider that for every quality differentiation degree* $\theta = \dfrac{s_1}{s_2} \in (0,1)$ *of demand system 3, firms 1 and 2 cartilize the market with a collusive price-setting strategy. Antitrust enforcement affects negatively and uniformly on the stability of the cartel, i.e., antitrust enforcement does not strongly affect any degree of product quality differentiation.*

Proof. To demonstrate this conjecture, it is enough to analyze the incentive compatibility constraint of the firm that produces low-quality products (see simulation in Section 6). □

Proposition 5.3 shows the antitrust enforcement affects negatively and in a uniform way on the stability of the cartel, that means that the first effected are the cartels that have highly differentiated and almost homogeneous products. This result guides the activity of the antitrust authority in the sense of focusing its research on potential cartels with slightly differentiated products because they are more stable. Thus, to completely deter cartels, the antitrust authority would have to focus its investigation on cartels with moderately differentiated products because a simple activation of a competition law is not enough to deter.

6 Numerical Simulation

Here, we did a numerical simulation on the stability of the cartel under different levels of antitrust enforcement — varying the fine rate levels and the levels of probability of detection — at different levels of product quality differentiation. For this purpose, we simulate the incentive compatibility constraint of the most sensitive firm because the cartel is sustained at the maximum critical discount factor of all participants. By Proposition 4.3, it is observed that the most sensitive firm is firm 1 and maintains the stability of the cartel with the following incentive compatibility constraint:

$$\bar{\delta}_1(\theta,\beta,f) = \frac{\pi_1^D - (1-\beta f)\pi_1^C}{(1-\beta)[\pi_1^D - \pi_1^N]}. \tag{24}$$

The double entry Tables have the following description: each Table has a description with the level of the fine rate, represented by f — imposed by the antitrust authority; in the horizontal heading are the different product quality differentiation degrees; on the left vertical side are the cartel probability detection levels, represented by the parameter β; and each in each box that is the intersection between product differentiation degree and the probability of detection is the critical discount factor $\bar{\delta}_1$ that maintains the stability of the cartel. To measure the effect of antitrust enforcement, we reference the first line of each Table of results — critical discount factors that support the cartel in the absence of antitrust enforcement. The interpretation of the double-entry Tables consists of: (i) if in each column the value of each box is greater than the base value, then antitrust enforcement harms the stability of the cartel; (ii) if the box is covered with the sign "-", the collusive agreement or cartel was completely dissuaded — there is no longer any collusive agreement.

Table 1: Simulation of the stability of the cartel at different levels of antitrust enforcement with a minimum fine rate $\bar{f} = 0.01$

β	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.8379	0.9237	0.9546	0.9705	0.9802	0.9868	0.9915	0.9950	0.9978
0.02	0.8551	0.9426	0.9741	0.9904	-	-	-	-	-
0.04	0.8729	0.9623	0.9944	-	-	-	-	-	-
0.06	0.8915	0.9827	-	-	-	-	-	-	-
0.08	0.9109	-	-	-	-	-	-	-	-
0.10	0.9312	-	-	-	-	-	-	-	-

Three Tables are simulated, each one with different levels of fine rate: Table 1 has a minimum fine rate $\bar{f} = 0.01$; Table 2 has a medium fine rate $\bar{f} = 0.105$; Table 3 has a maximum fine rate $\bar{f} = 0.2$.

The first rows of double-entry Tables 1, 2, and 3 show the levels of critical discount factors that maintain the cartel's stability in the absence of antitrust enforcement. These rows are the basis for measuring the effects of antitrust enforcement on the stability of the cartel. The three Tables show that when the degree of product differentiation is in the interval $[0.50, 1)$, the cartel is deterred at a level of probability of detection of $\bar{\beta} = 0.10$. Besides, they show that when the probability of detection is in $[0, 0.10]$, the cartel remains stable when the degree of product quality differentiation is $\theta = 0.10$.

Table 2: Simulation of the stability of the cartel at different levels of antitrust enforcement with an average fine rate $\bar{f} = 0.105$.

β	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.8379	0.9237	0.9546	0.9705	0.9802	0.9868	0.9915	0.9950	0.9978
0.02	0.8554	0.9428	0.9742	0.9904	-	-	-	-	-
0.04	0.8736	0.9626	0.9946	-	-	-	-	-	-
0.06	0.8925	0.9832	-	-	-	-	-	-	-
0.08	0.9123	-	-	-	-	-	-	-	-
0.10	0.9330	-	-	-	-	-	-	-	-

The double entry tables 1, 2, and 3 provide the same destabilization dynamics of the cartel, when the levels of antitrust enforcement are increased, the instability of the cartel increases uniformly, but the difference between them is that the levels of discount factors critics who still support the cartel are different. Therefore, we highlight that antitrust enforcement does negatively affect the stability of the cartel but does not affect any cartel with a specific product quality differentiation degree with greater intensity.

Finally, this result differs with the effect of antitrust enforcement on the stability of the cartel with horizontally differentiated products. We shows that antitrust enforcement has

Table 3: Simulation of the stability of the cartel at different levels of antitrust enforcement with a maximum fine rate $\bar{f} = 0.20$

β	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.8379	0.9237	0.9546	0.9705	0.9802	0.9868	0.9915	0.9950	0.9978
0.02	0.8557	0.9429	0.9743	0.9905	-	-	-	-	-
0.04	0.8742	0.9629	0.9948	-	-	-	-	-	-
0.06	0.8935	0.9837	-	-	-	-	-	-	-
0.08	0.9137	-	-	-	-	-	-	-	-
0.10	0.9347	-	-	-	-	-	-	-	-

a greater effect on the stability of cartels when the products are highly differentiated. In contrast, we show that when products are vertically differentiated, antitrust enforcement does not affect the stability of the cartel with greater intensity according to the product differentiation degree.

7 Conclusion

This paper examines three aspects of a cartel made up of a duopoly with a collusive price-setting strategy in a market with vertically differentiated products — product differentiated by quality. First, it diagnoses which of the participating cartel firms is more susceptible to destabilizing the cartel. Second, it analyzes the trade-off between the degree of product quality differentiation and the sustainability of the cartel. Third, it examines the effect of antitrust enforcement on cartel stability at different levels of product quality differentiation.

On one hand, we show that for any degree of product quality differentiation, the firm that produces low-quality products (firm 1) is more susceptible to destabilizing the cartel than the firm that produces high-quality products (firm 2), which implies that the stability of the cartel depends on more about the firm 1. Besides, we show that the trade-off between the degree of product quality differentiation and the sustainability of the poster has a quadratic behavior, that means, that when the products are highly differentiated and almost homogeneous, the cartels are less stable. On the other hand, we show that for every product quality differentiation degree, antitrust enforcement has a uniform effect on the stability of the cartel, this means that antitrust enforcement negatively impacts the stability of the cartel but does not impact with greater intensity on some specific degree of product quality differentiation.

This topic serves as a guide for antitrust policymakers and antitrust authority activity to focus their research on potential cartels based on the product quality differentiation degree. Besides, it helps to identify which of the participating firms of a potential cartel can be urged to expose their illegal activity.

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