Lucas' overstatement: measuring the real effect of human capital on growth through an institutional approach

Fabio Gama
PhD student at PPGE / UFJF
Federal University of Juiz de Fora
Juiz de Fora, MG, BRAZIL 36036-900
f140383@gmail.com

Suzana Quinet de Andrade Bastos (Corresponding author)
Professor at PPGE / UFJF
Cnpq Productivity Scholarship
Federal University of Juiz de Fora
Juiz de Fora, MG, BRAZIL 36036-900
quinet.bastos@ufjf.edu.br

Tiana Assis
Undergraduate in Economics at UFJF
Federal University of Juiz de Fora
Juiz de Fora, MG, BRAZIL 36036-900
tianaweinmann@gmail.com

Resumo: Este artigo propõe uma reinterpretação do modelo de crescimento endógeno de Lucas (1988), acrescentando o componente institucional como um dos seus determinantes. Seguindo Acemoglu (2014), avaliamos o crescimento econômico através de seus determinantes aproximados e fundamentais. Para tanto, desenvolvemos um modelo em que o nível institucional atua como impulsionador do efeito do capital humano sobre o produto, sendo testado empiricamente para 40 países nos anos 2000, 2005 e 2010. Os resultados encontrados corroboram a teoria institucionalista; verificamos que o modelo de Lucas superestima a contribuição do capital humano no crescimento. Além disso, as evidências também indicam que o capital humano é, de fato, impulsionado institucionalmente e funciona como um canal para as instituições.
Palavras-chave: crescimento endógeno; capital humano; instituições

Abstract: This paper proposes a reinterpretation of Lucas endogenous growth model (1988), by adding the institutional component as one of its determinants. Following Acemoglu (2014) we evaluate the economic growth through its approximate and its fundamental determinants. For such, we develop a model in which the institutional level performs as a booster for the effect of human capital on the product and test it empirically for 40 countries in the years 2000, 2005 and 2010. The results we found support the institutionalist theory; we verify that Lucas' model overestimates the contribution of human capital on growth. In addition, evidence also indicates that human capital is, in fact, driven institutionally and works as a channel for institutions.
Keywords: endogenous growth; human capital; institutions
JEL: E02, O43, O11, C33, C36

Crescimento Econômico e Desenvolvimento Regional

1. INTRODUCTION

The causes involved in promoting long-term economic growth have always been of interest in economic sciences. Since eighteenth and nineteenth centuries important economists as Adam Smith (1776) and David Ricardo (1817) were bringing contributions to the understanding of what economic growth and some of the mechanisms for reaching it would be. From the 20th century, the discussion is replaced by large contributions obtained through formalization theory in modelling. The Harrod-Domar model, developed independently by the economists Roy F. Harrod (1939) and Evsey Domar (1946), demonstrates that the rate of economic growth would be directly related to the savings and labor power level present in the economy. It is still expected that long-run balance could not be achieved since it would not be possible to establish guarantees in relation to investment level performed in order to achieve continuous expansion of economic activity because the agents operate under expectations (Harrod 1939; Barcelos & Salles 2011).

In 1956 the American economist Robert M. Solow criticized and proposed refinements to the theoretical construction of Harrod and Domar, thus launching what became known as the neoclassical model. In this, Solow introduced the contribution of technology to economic growth and assumed that this factor and the labor force grow at constant rates, both given exogenously. In opposition to the Harrod-Domar model, Solow abandoned the capital and labor condition that cannot be allocated to each other in production, which allowed the adjustment capacity between these two factors in a state of continuous and balanced long-run growth. Solow's model (1956) also implied that all countries - with identical preferences and technological level - would be in a movement of convergence in relation to their growth rates and income level; and that the poorest economies would present an accelerated growth rate due to its lower labor-capital ratio and therefore its higher marginal product.

From the 1980s new contributions are made to the theory. Committed to clarify the differences between the empirical facts of economies and the Solow model, Paul Romer (1986) and Robert Lucas (1988) inaugurated the studies of the economic growth determinants through an endogenous understanding of its variables and the addition of human capital to their models. In the new model, human capital takes the form of education, health and experience, presenting positive externalities and increasing returns in productivity. With these characteristics, the human capital factor becomes capable of increasing the productivity of physical capital and labor, thus fostering economic growth over time. Thereon, unlike Solow, Romer assumes that it is in fact possible for larger economies to grow faster than the less advanced ones; which could justify the empirical findings where high average GDP growth rates were noted in richer countries such as the USA and Western Europe while a very modest one was coming from the poorest countries such as those located in Central America.

The theoretical construction developed by the endogenous growth studies deserves great merit. However, we can identify that it does not shed some real light upon what would be the determinants of growth/accumulation of the human capital factor – which is a great issue since this one is the major driver of the economic growth in the model. One of the main contemporary authors discussing the role that the institutions of a country have as promoters of sustained economic growth is Acemoglu. In his work, it is emphasized the importance that institutions have in this process, evidencing their capacity before the agents to create and assure the propitious environment to the economic growth (Acemoglu et al. 2005).

According to Acemoglu et al. (2014) – and following North & Thomas (1973) -, the economic growth determinants would be expressed by approximate determinants and fundamental determinants. Factors of production, innovation, and accumulation of physical and human capital would be included in the approximate determinants; albeit, the origin of nations' success and their differences would not be verified, but prosperity itself. The fundamental determinants would be the real responsible for the success origin and these would be the countries institutions. Institutions lead, raising the level of productivity, human capital and physical capital, which reproduce economic growth.

Therefore, the current work seeks, based on the institutionalist theory, to internalize in the Lucas Model (1988) the quality of institutions as the main determinant for variation in the stock of human capital and, consequently, the driver of long-run economic growth in the countries. Through Pooled Ordinary Least Squares estimation (POLS) and Instrumental Variables (IV) we compare the Lucas growth model to the Lucas extended model; the last one obtained by inserting institutions as a human capital stock booster.

As foreseen by institutionalist authors, the level of economic and political institutions of a country plays a key role in economic growth through human capital. Hence, Lucas Model overestimates the contribution of human capital since in the presence of institutions there is a reduction in human capital-GDP per capita elasticity. Other evidence found reinforces the correlation between the institutional variable and human capital, when is noted that in the Extended Model the effect of physical capital remains statistically unchanged if compared to the same effect estimated through Lucas Model. Therefore, the results also indicate that, essentially, institutions are able to establish an influence channel through human capital in the countries growth process.

The study proceeds as follows. Section 2 discusses the theory that relates institutions and economic growth. Section 3 presents the model. Section 4 addresses the empirical model and database. Then, in section 5, the results of the empirical application of the model are displayed. And finally, section 6 concludes the paper.

2. INSTITUTIONS AND ECONOMIC PERFORMANCE

The institutionalist theory has as initial mark the contributions of Douglass North (1993). The author points to an effective economic organization as the crucial factor for economic growth. For North, the real growth consists in the raising of the population income per capita level, which would remain at a constant level- or at steady state - during periods when individuals could not obtain sufficient incentives to develop activities that assist economic growth in society. According to North & Thomas (1973, p. 2): " (innovation, economies of scale, education, capital accumulation, etc.) are not causes of growth; they are growth." Hence, the author characterizes institutions as a channel through which human - and physical - capital act to result in economic growth, thus participating as means and end in this process; and also highlights the presence of *path dependency,* to compare the economic performance among nations - following the quality of its institutions - to North: "history matters" (North 1990, p. 100)[1].

Acemoglu et al. (2014) in its approach on the determinants of economic growth highlighted its approximate and fundamental determinants - following North & Thomas (1973) – pursuing to answer questions such as: "Why some countries invest more in their education system? " Or "Why do people save and invest in physical capital?" Recognizing institutions as the fundamental determinants, the authors defend that growth with raising productivity, human capital, and physical capital would be the outcome of how these institutions evolve over time.

Acemoglu et al. (2005) argue about the importance of institutions in shaping economic incentives of key players in society, as in cases of incentives to investments in human and physical capital, and technology. For the authors, the disparities in economic growth verified in nations are dictated by the interaction among critical circumstances - as major historical

[1] North & Thomas (1973) contemplate several historical events which took place in the Western world, allowing institutional arrangements, over time, to evolve and result into new forms of social organization of peoples - and in their economic prosperity.

events - and the institutional characteristics of these countries. Such critical circumstances[2] can be seen as random events - or shocks- which strike societies and come demanding an institutional response. In these cases, pre-existing institutions[3] as they relate to such events, will move towards a differentiation that, over time, will result in their superior or inferior economic performance vis-à-vis other nations (Acemoglu & Robinson 2012, p. 113).

Aiming to shed light on what would make institutions good, i.e., more suited to promoting the countries' economic success, Acemoglu et al. (2005) bring in the term "inclusive institutions", which, unlike "extractive institutions", are organized in a politically centralized model and allow society to have access to the available resources. Therefore, such institutions serve the purpose of the majority and not the interests of a small elite - some restricted group[4]. When political centralization is not followed by economic pluralism, institutions still shape themselves as politically extractive, and economies can only achieve some degree of prosperity - even significant - but this growth momentum is not able to be prolonged for long periods[5]. Then, a scenario of innovation deprivation is established, leading to the collapse of the system. According to Acemoglu & Robinson (2012, p. 136): "But as in all instances of growth under extractive institutions, this experience did not feature technological change and was not sustained. Growth first slowed down and then totally collapsed".

It is the political institutions present in countries - whether extractivist or not - that usually dictate which model of economic institution will prevail (Acemoglu & Robinson 2012). According to Acemoglu, this dynamic can be understood by using the "vision of social conflict" developed in North (1981). According to this, the power of choice over the institutions - political and economic - that will prevail in a country is not concentrated, at all times, in the hands of the majority of citizens, but rather of an elite. This group will use these same institutions to achieve a higher level of well-being, even if this implies underdevelopment for the rest of the population (Acemoglu et al. 2005). Thus, in the midst of the temporal interest game in which different parts of society act for their own benefit, the institutional apparatus will eventually be constructed and directed in favor of individuals who already hold some power in this provision.

Acemoglu et al. (2012) address the distribution of political power in society distinguishing between the two types of powers that are established in social relations: *de jure* political power and *de facto* political power - that are originated differently. The first one refers to the power derived from the institutions, i.e., established by them. Acemoglu et al. (2005, p.7) emphasize: "For example, in a monarchy, political institutions allocate all *de jure* political power to the monarch, and place few constraints on its exercise. " The second refers to the form of political power attributed to certain individuals even though in a way not foreseen by the institutions - unconstitutionally. The political power, *in fact*, would derive, then, from the economic resources that given parts of society possess. In such a way, these groups could be able to influence prominently the direction institutions of a country would follow, if they simply used their wealth, activating mechanisms - such as protests, weapons, army cooptation, among others - in order to trigger changes at the institutional level in support of their own agenda.

De facto power shows to be responsible for pointing the direction in which political institutions will move, and these, therefore, determine the configuration in which *de jure* power

---

[2] The authors exemplify the 14th century Black Death as one of these critical events present in history.

[3] Pre-existing institutions are defined by various factors such as cultural and historical elements, social relations, and random events.

[4] Political centralization is necessary for the establishment of law and order, thereby ensuring the fundamental guarantees a nation must possess in order to offer incentives to achieve its economic prosperity.

[5] The main cause for such argument is the lack of creative destruction - not promoted due to the fear those in power have to shake the *status quo* in which society is already settled. Responsible for the technological advancement, therefore, in its absence, any level of sustainable growth is constrained.

should be exercised in social relations. Resource allocation is therefore imperative for understanding the determinants of economic growth. The authors also highlight the endogeneity presented by political and economic institutions in which: "Finally, political institutions are also endogenous; the current balance of political power, incorporating both *de jure* and *de facto* elements, also determines future political institutions." (Acemoglu et al. 2005, p. 451). Thereby, *de jure* and *de facto* political power in a period *t* will influence the choice of the economic institutions of the same period and the follow-up that will be given to the political institutions in a near period (*t + 1*). The economic institutions resulting from this choice process, in turn, will be determinants in the economic growth of the period *t* and on the distribution of resources for the period *t + 1*.

Inclusive economic institutions do not have the ability to support or to offer support to political extractive institutions, because if this happens, those economic institutions also become extractivist; or will lead to a dynamic of transforming political institutions before extractive into inclusive (Acemoglu & Robinson 2012). The coherence contained in the conciliation between inclusive institutions is sustained by the same factor responsible for the stagnation of economic growth bound to the countries oriented by extractive political or economic institutions: the creative destruction. The authors describe that the "controlled stagnation" some agents enable in the economies is intended to assure there will be fruits to be harvest since the best ones are reserved to those holding power. Acemoglu also highlights:

> Some ways of organizing societies encourage people to innovate, to take risks, to save for the future, to find better ways of doing things, to learn and educate themselves, solve problems of collective action and provide public goods. Others do not. (2005, p. 397)

3. THE MODEL

Assuming a Cobb - Douglas production function, the Lucas Model (1988) recognizes the economic activity level of the countries as a function of physical capital, human capital and a set of constant factors in time. In formal terms, equation (1) summarizes the model central concept:

$$Y_t = Ak_t^{\beta}(u_t h_t)^{1-\beta} \qquad (1)$$

In that, *Y* is the economic activity level of a given country in period *t*. *A* is a constant, *k* and *h* are, respectively, accumulated physical capital and human capital in a given period, and *u* is the share of human capital destined to goods and services production. Despite Lucas' progress in relation to the predecessor models, this one did not discuss the role that the institutions would have in the level of activity. According to Lucas, institutional differences in the level of activity would be captured within the constant model parameter, so that its effects on the product over time would not be modelled.

Based on Acemoglu et al. (2014), this paper suggests a new version of Lucas Model (equation 2), in which institutions are added as boosters for human capital, thus assuming their function as a fundamental determinant of growth. The hypothesis is that the institutional level enhances the effect of human capital in the level of production[6]. Equation 2 expresses how this relationship is established[7]:

$$y = Ak_t^{\beta}(u_t h_t I_t)^{1-\beta} \qquad 0 < \beta < 1 \text{ and } 0 < u_t < 1 \quad (2)$$

[6] Solow (1956) demonstrates the role of technology in making work more efficient in the production function.
[7] We note this positive effect by encouraging a greater interaction among workers - via improved social capital, for example -, thereby, institutions produce greater externality of knowledge in the production process.

In (3) I stands for the institutional level of the country in period *t*.

Following Lucas (1988), in order to find a model solution, we make use of the utility maximization problem faced by agents, subject to temporary restrictions. The representative agent maximizes the present value of the utility flow of consumption, *U(Ct)*, throughout its life, discounted by its intertemporal preference ρ rate[8]. The agent utility function may be given by:

$$U(Ct) = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} \qquad (3)$$

Where σ is a risk aversion coefficient, we assumed $\sigma > 0$.

Moreover, in each period of time, the agent seeks to maximize its intertemporal utility, subject to the following restriction:

$$\dot{k} = Ak_t^\beta (u_t h_t I_t)^{1-\beta} - C \qquad (4)$$

Equation 4 reports that the variation of the capital stock is the output of the period minus consumption, i.e., investment equals saving.

$$\dot{h} = \theta h(1 - u_t) \qquad (5)$$

With $\theta > 0$

In the accumulation of knowledge (5), agents learn when they study. Therefore, the accumulation of human capital is related to time spent away from work $(1 - u\ )$.

$$I_t = Je^{nt+\sum_{\tau=1}^{t} \varepsilon_\tau} \qquad (6a)$$

Finally, the institutions of period *t* are an exponential function of a *n* constant, and shocks, ε, which accumulate in time, (6a). However, Acemoglu et al. (2005) argue that shocks occur randomly in the process of the institutions evolution, i.e., they can be positive or negative and do not follow a predetermined pattern. Thus, we assume that, in the limit, the sum of the shocks is zero. Thereon, rewriting 6a, we have 6b.

$$I_t = Je^{nt} \qquad (6b)$$

Therefore, the Hamiltonian of current value, L, is given by:

$$L = \int_1^\infty e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} + V\left\{Ak_t^\beta (u_t h_t Je^{nt})^{1-\beta} - C\right\} + \lambda\{\theta h(1 - u_t)\}$$

The first order conditions are as follows:

$$\frac{\partial L}{\partial u} = V\left\{Ak_t^\beta (1-\beta)(h_t Je^{nt})^{1-\beta} u_t^{-\beta}\right\} - \lambda\theta h = 0 \qquad (7)$$

$$\frac{\partial L}{\partial c} = e^{-\rho t} c_t^{-\sigma} - V = 0 \qquad (8)$$

$$\frac{\partial L}{\partial k} = V\left\{\beta Ak_t^{\beta-1} (u_t h_t Je^{nt})^{1-\beta}\right\} \qquad (9)$$

$$\frac{\partial L}{\partial h} = V\left\{(1-\beta)Ak_t^\beta (u_t Je^{nt})^{1-\beta} h_t^{-\beta}\right\} + \lambda\{\theta(1 - u_t)\} \qquad (10)$$

Transversality conditions

$$\frac{\partial L}{\partial k} = -\dot{V} \qquad (11)$$

$$\frac{\partial L}{\partial h} = -\dot{\lambda} \qquad (12)$$

$$\frac{\partial L}{\partial u} = 0 \qquad (13)$$

[8] As lower the discount ρ factor is, the agent values more future consumption over current consumption.

Solving it, we have the growth rates of consumption, capital stock, human capital and product, respectively

$$\frac{\dot{c}}{c} = \gamma = \frac{\theta-\rho}{\sigma} + \frac{n}{\sigma}$$

$$\frac{\dot{k}}{k} = \gamma = \frac{\theta-\rho}{\sigma} + \frac{n}{\sigma}$$

$$\gamma_h = \frac{(\theta-\rho)}{\sigma} + n(\frac{1}{\sigma} - 1)$$

$$\frac{\dot{y}}{y} = \beta\left[\frac{\theta-\rho}{\sigma} + \frac{n}{\sigma}\right] + \left[(1-\beta)\left\{\frac{(\theta-\rho)}{\sigma} + n(\frac{1}{\sigma} - 1)\right\}\right] + (1-\beta)n \tag{14}$$

Similarly to Lucas Model, we note that if the risk aversion coefficient, σ, is high, then the growth rate of agent consumption is lower, having a negative impact on the growth rate of the economy product. On the other hand, if the intertemporal discount rate is small, the higher the rate of economic growth. This because, if agents are valuing future consumption more, the greater the savings. In addition, the larger the θ of the human capital employed in the production of more human capital, the higher the growth rate. Lastly, leaving aside the similarities with the antecedent model and moving towards the effects of institutions on growth (equation 2), according to equation 14, we can note that the higher the growth rate of institutions over time, n, the higher the growth rate of product.

4. EMPIRICAL MODEL AND DATABASE

Equation 2 attests that the level of activity, Y, of a country is a function of physical capital (k), human capital (H) and its institutions (I). Therefore linearizing 2, and inserting an idiosyncratic error term, ε, we obtain the empirical model to be tested:

$$Y_{i,t} = \beta_0 + \beta_1 K_{i,t} + \beta_2 H_{i,t1} + \beta_3 I_{i,t} + \varepsilon_{i,t} \tag{15}$$

The proposition behind the hypothesis of Lucas (1988) consists of the higher the level of physical capital and human capital, the more efficient the productive process becomes, reflecting an increase in the product/worker ratio. Also, we expect that institutions influence positively the level of activity once, according to Acemoglu et al. (2005), better political and economic institutions amplify the effect of human capital on the level of activity.

For the equation 15 estimation, we use a set of variables comprising more than 40 countries in a series of three-time cuts in the years 2000, 2005 and 2010. The variables aim to capture the levels of activity, physical capital, institutions and human capital of nations.

As a proxy for activity level, World Bank data are used for GDP *per capita* (*GDPpc*)[9], which is defined as gross domestic product divided by the middle-aged population.

The variable Gross Fixed Capital Formation (*FBKF*) of the World Bank includes land improvements; plant, machinery and equipment purchases; and construction of roads, railways and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings.

Given the lack of uniformity in the literature regarding the optimal proxy for human capital, a human capital index is built (lnhumancap) via Multivariate Analysis (MA). Based on Becker (1994), Barro (2001), Cervellati & Sunde (2005), and Tuna et al. (2007)[10], we selected the variables - also collected from the World Bank - of education, "Average number of years of total schooling across all education levels (population aged 15-64)"; health, "Life expectancy

[9] GDP per capita at constant 2010 prices.

[10] The authors whether discuss or test how these variables are related to human capital and contribute to economic growth.

at birth (total population)"; high-level technology production, "Information and communication technology service exports"; and inventive production, " Patent applications by residents in national offices ", for the MA implementation for each year of the sample (2000, 2005 and 2010).

Finally, we use the overall score of the Index of Economic Freedom produced by the *Heritage Foundation*[11] as a proxy for Economic and Political Institutions. With scores ranging from 0 to 100, it focuses on four main aspects of countries' economic environment, which are usually influenced by government action: Rule of Law, Government Size, Regulatory Efficiency, and Market Openness. To evaluate such aspects, the index measures 12 components capable to capture, jointly, the economic and political institutional quality of countries. These components are Property Rights, Judicial Effectiveness, Government Integrity, Tax Burden, Government Spending, Fiscal Health, Business Freedom, Labor Freedom, Monetary Freedom, Trade Freedom, Investment Freedom and Financial Freedom.

Therefore, we can assume a direct relation between scores and the countries' institutions when a higher score reports also a higher institutional level.

5. RESULTS

In the Multivariate Analysis, we selected only one factor in each *cross-section*, which represents 78%, 94% and 100%[12] of the sample variation, respectively[13]. Appendix 2 presents the ranking in ascending order of the countries, according to the MA *scores*. In general, we note that countries with higher *scores* - best ranking in human capital ratio - are those with a higher level of economic growth.

With the MA results, we estimate the Extended Model of Lucas, 15, and the original Lucas Model via panel data[14] in order to compare its empirical implications. The first column of table 1 shows the results of the estimation of equation 15 by POLS[15]. Regarding the model fit quality, R^2 indicates that 78% of GDP per capita variation is explained by independent variables. The coefficients, the parameter that measures the effect of physical capital (lnfk), show to be positive and significant at 99% confidence, reporting that a 1% increase in physic capital leads to an increase of 0.13% in the level of economic activity. Following Lucas (1988), the greater the stock of available physical capital, the greater the worker's productivity, reflecting positively on the product.

[11] The institutional data used in this study can be found at https://www.heritage.org/index/

[12] According to Hair (2005) in social sciences, in which information is generally less accurate, it is plausible to consider a solution that explains 60% of the total variance (and in some cases even less) as satisfactory. Under this argument, it was decided to use the cumulative 75% explanation of the original variables, which covers only the first factor of each year.

[13] Appendix 1 displays the results of Multivariate Analysis with the Factor Composition, and Kaiser-Meyer- Olkin (KMO) statistics for the MA in each year. The KMO statistic is used to test the consistency of the MA application, its values are between 0 and 1. Values of 0.80 or more are considered admirable; between 0.70 and 0.80, medium; between 0.60 and 0.70, mediocre; between 0.50 and 0.60, bad; and below 0.50, unacceptable. For the three years, the variables with the greatest explanatory power of the data set variation are education and life expectancy, in addition, KMO statistics were above 0.60.

[14] It is worth mentioning, in order to obtain the results in terms of percentage variations, the variables were logarithmized. Due to the logarithmization, it was appealed to transforming the Multivariate Analysis scores in order to generate non-negative values. The transformation sought to maintain ordering and the magnitude difference from the original scores. Thus, for each year, it was selected the lowest *score*, $score_{min}$, and obtained the difference of *scores* in the *n* observations in relation to the $score_{min}$. In formal terms, the new value for *n*-observation, $score_n^*$, is given by: $score_n^* = score_n - score_{min}$, with $n = 1,2,4,\ldots 40$.

[15] The estimation by POLS consists of stacking the observations in a single cross-section and then estimating the parameters by Ordinary Least Squares (OLS). For more details, see Wooldridge (2002).

TABLE 1 - Regression - Dependent Variable: Log GDP *per cap*

	POLS		IV	
Model/variables	Extended Model (1)	Lucas (1988) (2)	Extended Model (3)	Lucas (1988) (4)
lnfk	0.13***	0.13***	0.10**	0.09*
	(0.002)	(0.011)	(0.025)	(0.086)
lnhumancap	1.10***	1.53***	1.66***	1.92***
	(0.000)	(0.000)	(0.000)	(0.000)
lnoverall	3.061***	-	1.53*	-
	(0.000)	-	(0.076)	-
Constant	-6.99***	5.59***	-0.05	6.31***
	(0.001)	(0.000)	(0.988)	(0.000)
Observations	117	117	118	118
R^2	0.78	0.71	0.78	0.71

*** p <0.01, ** p <0.05, * p <0.1
† Standard errors in parentheses
Source: Own elaboration

The variable that measures the role of human capital over the product (lnhumancap) proves to be positive and significant at 99% confidence. Thus, an improvement of 1% in the population knowledge, produces on average a 1.10% increase in the product. According to Lucas (1988), to the extent that workers have more knowledge, they become more productive, resulting in increased production.

These results are widely discussed and consolidated in the literature by Lucas's seminal work (1988). The current work contribution arises, therefore, when inserting the role of the institutions in the determination of the product. As can be seen, in column 1, the coefficient that measures the effect of institutions on the countries' output (lnvorverall) is positive and significant at 99%, allowing to assert that an improvement of 1% in institutions produces a positive effect of 3.06% in GDP. This result corroborates the findings of the model (equation 14) used, which indicates that the level of economic activity and institutions correlate positively.

In column 2, this study proceeds with the estimation of the original Lucas Model (1988) by POLS, which measures the effect of the physical (lnfk) and human capital (lnhumancap) on the product (lngdppc). The purpose of this exercise is to compare the estimation of the original model (1988) with the one obtained by the Extended Model - Equation 2. That is to verify how the significance and magnitude of the original model parameters (1988) behave after the institutional variable is inserted. R^2 indicates that 71% of GDP per capita variation is explained by the variation of the exogenous variables. In addition, the coefficients obtained are positive and significant at 99%. Hence, a positive change of 1% in physical capital and human capital raises the product level by 0.13% and 1.53%, respectively[16].

Comparing the estimates of columns 1 and 2, it can be highlighted the two main findings. The first one comprehends the difference between the adjustment degree of the models obtained by inserting the institutions in Lucas Model, the explanatory power of the variability of GDP per capita, R^2, raises 7 percentage points. The second one is related to the magnitude of the estimated parameters. To the extent that political and economic institutions are recognized as determinants for activity level and inserted in Lucas Model, the magnitude of the parameter related to physical capital remains constant, albeit, the effect of human capital on GDP *per capita* is considerably reduced, declining from 1.53 to 1.10.

Nevertheless, although the empirical evidence indicates a good fit of the Extended Model to express the correlation between the level of activity and the explanatory variables - physical capital, human capital and institutions – we can assume that the results estimated by POLS do not reflect a very accurate correlation between the explanatory variables and the dependent variable. According to Acemoglu et al. (2014), a potential issue when estimating the relation between human capital and economic growth by OLS arises from the inconsistency generated by the causality among the studied variables. For the authors, despite the consensus in the literature indicates an effect that flows from human capital towards economic growth, it cannot be entirely reject that there is an effect in the opposite direction, i.e., that a more elevated income level could, through various channels, lead to a higher instructional level.

In order to correct this possible bias, Acemoglu et al. (2014) suggest the use of Instrumental Variables (IV)[17]. Thereon, the authors use the primary school enrollment rate of the late nineteenth century, more specifically, the year 1870, to instrumentalize the human capital proxy of years of study in 2005. So, in order to make the correlation predicted by POLS robust, the exercises in columns 1 and 2 are repeated using IV. Based on Acemoglou et al. (2014), we use as instrument for human capital proxy (lnhumancap) in the years 2000, 2005 and 2010, the primary school enrollment rate of the population aged 15 to 64 years in the end of the 19th century, in 1890, 1900 and 1910, respectively[18].

Columns 3 and 4 of Table 1 present the results obtained through IV estimations: Extended Model (equation 2) and Lucas Model, respectively. Similarly to the POLS analysis, for both models, the coefficients of the explanatory variables present significant and positive signs, indicating that positive variations in physical capital, human capital and institutions affect activity level positively. Moreover, in terms of adjustment and differential magnitude in the coefficients for the human capital variable proxy (Leduc), the results provide further evidence as produced in the POLS analysis, indicating that, once inserting the institutional factor in the Lucas Model, explanatory power rises about 19 points - 0.51 to 0.70 - and the coefficient which measures the effect of human capital on GDP *per capita* falls from 1.92 to 1.66.

In general, the evidences presented by the Extended Model and the Lucas Model, both by POLS and IV, corroborate Acemoglu et al. (2014), since, according to the authors, empirical models that do not control adequately the effect of institutions on the activity level tends to

[16] In order to verify the presence of endogeneity in the estimated models, a regression of the residues against the explanatory variables was applied (see Appendix 3). The results suggest no significance of the parameters, indicating the consistency of our estimations.

[17] The IV method predicts the two-stage estimation of the relation between the dependent variable and the explanatory variables. In the first stage, the relation between the endogenous explanatory variable with the other exogenous explanatory variables and the instrument is estimated – the Instrumental Variable that has a high correlation with the endogenous explanatory variable and no correlation with the error term. In the second stage, we estimate the relationship between the dependent variable and the other exogenous explanatory variables and the predicted values of the endogenous explanatory variable. For more details, see Wooldridge (2002).

[18] The variable containing primary enrollment rate for the population aged 15-64 was extracted from Lee & Lee (2016). In addition, in the first stage estimation, we use the logarithm of the primary enrollment rate – lnprm.

suffer from a serious bias of omitted variable. Therefore, the effect of human capital on the countries product measured by the original model (1988) is overestimated, as institutions and human capital are correlated factors.

6. CONCLUDING REMARKS

The present study was developed aiming to investigate the role that political and economic institutions play in promoting the human capital factor in the Lucas Model (1988). The discussion proposed by Acemoglu et al. (2014) and Acemoglu et al. (2012) on institutional quality being responsible for the variations in the level of human capital presented by the countries elucidates how the incentives and safeguards promoted by political and economic institutions foster the growth process via human capital.

The results of estimations via Pooled Ordinary Least Squares Estimation (POLS) and Instrumental Variables (IV) compare the Lucas growth model (1) to the same model of Lucas in an extended form (2); the last one obtained by inserting institutions as a human capital stock booster. As predicted by the authors, the level of a country's political and economic institutions plays a key role in economic growth via human capital; and it can be seen that the original Lucas Model overestimates the contribution of human capital in the growth process since it does not model institutions as one of its determinants.

Therefore, as discussed by Acemoglu et al. (2014), one can understand physical capital and human capital as proximate causes of economic growth. The comparative analysis between the Lucas Model (1988) and the Extended Model proposed in the present study aims to indicate the overestimation in the effect of human capital on the output produced by the first one, once it does not consider institutions and human capital as correlated factors; notwithstanding, according to Acemoglu et al. (2014), the human capital factor is determined institutionally and works as an influence channel on institutions.

As presented in this study, the discussion concerning the impact of institutional composition on the level of economic prosperity continues to be reinforced through the empirical findings. Relying on the analysis of Acemoglu et al. (2012), it can be pointed a robust and independent judiciary power as a central effort in order to assure institutions to be channels operating under what the *de jure* power states, self-oriented for the general benefit - and not under the prominent influence of individuals or small groups. The relation established between institutions and resources distribution in society, therefore, reinforces the constant need for policing the institutions and also suggests a better level of income distribution as an important mechanism in order to promote more inclusive institutions - which ones, ultimately, will lead to larger economic outputs experienced by countries.

This is one more effort in the area of institutional studies that aims to bring additional results when proposing an extension to the Lucas Model (1988). Remain as a recommendation for future research on the subject to adopt, as they become available, other variables for measuring human capital, which will bring more refining to the effect of institutions on the factor. As we have attempted to include a number of countries with great institutional diversity, we would also recommend to further researches to expand the sample of countries to be tested, continuing to consider institutional diversity as one of the analysis main concern.

REFERENCES

Acemoglu, D, Gallego, F, A, Robinson, J, A 2014, Institutions, human capital, and development, *Annual Review of Economics*, vol. 6, pp. 875-912. https://doi.org/10.1146/annurev-economics-080213-041119

Acemoglu, D, Johnson, S, Robinson, J, A 2005, Institutions as a fundamental cause of long-run growth, *Handbook of economic growth*, vol. 1, pp. 385-472. DOI: 10.3386/w10481

Acemoglu, D, Robinson, J, A 2012, *Why the Nations Fail: The Origins of Power, Prosperity, and Poverty*, Crown business, New York.

Atun, R, A, Harvey, I, Wild, J 2007, Innovation, patents and economic growth, *International Journal of Innovation Management*, vol. 11, no. 02, pp. 279-297. https://doi.org/10.1142/S1363919607001758

Barcelos, G, L, Salles, A, O, T 2011 The Post-General Theory debate between Keynes and Harrod and the emergence of dynamic analysis in economic science, paper presented at the *National Meeting of Economy-ANPEC*, no. 39.

Barro, R, J 2001, Human capital and growth. *American economic review*, vol. 91, no. 2, pp. 12-17. DOI:10.1257/aer.91.2.12

Becker, G, S 1994, Human capital revisited. In: *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*, pp.15-28. The University of Chicago Press.

Cervellati, M, Sunde, U 2005, Human capital formation, life expectancy, and the process of development, *American Economic Review*, vol. 95, no. 5, pp. 1653-1672. DOI:10.1257/000282805775014380

Harrod, R, F 1939 An Essay in Dynamic Theory, *The Economic Journal*, vol. 49, no. 193, pp. 14-33. https://doi.org/10.1111/ecoj.12224

Lee, J, W, Lee, H 2016. Human Capital in the Long Run, *Journal of Development Economics*, vol. 122, pp. 147-169. https://doi.org/10.1016/j.jdeveco.2016.05.006

Lucas, R, E, Jr 1988. On the mechanics of economic development, *Journal of monetary economics*, vol. 22, no.1, pp. 3-42. https://doi.org/10.1016/0304-3932(88)90168-7

North, D, C, Robert T 1973, *The Rise of the Western World: A New Economic History*, Cambridge University Press.

North, D, C 1981, *Structure and change in economic history*, W. W. Norton & Company.

North, D, C 1990, *Institutions, Institutional Change and Economic Performance*, Cambridge University Press.

Romer, P, M 1986, Increasing returns and long-run growth, *Journal of Political Economy*, vol. 94, pp. 1002-1037. https://doi.org/10.1086/261420

Solow, R, M 1956, A contribution to the theory of economic growth, *The Quarterly Journal of Economics*, vol. 70, no. 1, pp. 65-94. https://doi.org/10.2307/1884513

APPENDIX

Appendix 1 - Correlation of Variables with KMO Factors and Statistics

Year	2000		2005		2010	
Variables \ Factor	Factor 1	Factor 2	Factor 1	Factor 2	Factor 1	Factor 2
educ	0.6649	0.2581	0.6272	0.2577	0.6604	0.2163
lexp	0.6668	0.2322	0.6065	0.1825	0.6196	0.1221
pa	0.2580	0.5406	0.2983	0.4213	0.2245	0.3728
ict	0.3473	0.5484	0.3461	0.4251	0.3592	0.4048

KMO	0.6730	0.6730	0.6262

Source: Own elaboration

Appendix 2 - Factor Analysis *Scores* for the years 2000, 2005 and 2010

2000		2005		2010	
Countries	Score	Countries	Score	Countries	score
South Africa	-1.588526	South Africa	-1.645458	South Africa	-1.523305
India	-1.537812	Bangladesh	-1.271994	Bangladesh	-1.296017
Bangladesh	-1.45834	India	-1.243655	Morocco	-1.222189
Morocco	-1.317865	Morocco	-1.199067	India	-1.20314
Egypt	-0.9284954	Egypt	-0.9198097	Egypt	-1.035694
Thailand	-0.8107104	Philippines	-0.7742772	Philippines	-0.8919634
Philippines	-0.7545543	Thailand	-0.6414008	Tunisia	-0.6285537
Brazil	-0.7312374	Colombia	-0.6362991	Brazil	-0.6089301
Colombia	-0.6093456	Brazil	-0.6025319	United States	-0.6084934
Tunisia	-0.5872911	Tunisia	-0.5798982	Thailand	-0.5697126
United States	-0.4129152	United States	-0.4984709	Colombia	-0.4553139
China	-0.384685	China	-0.2936164	Peru	-0.4467457
Peru	-0.3084984	Russia	-0.2746321	China	-0.4272388
Russia	-0.2480756	Uruguay	-0.2534959	Uruguay	-0.3842694
Mexico	-0.2000707	Peru	-0.2193198	Mexico	-0.3034097
Bulgaria	-0.1147114	Mexico	-0.2089145	Argentina	-0.2151873

Uruguay	-0.1027735	Portugal	-0.1665274	Portugal	-0.186023
Jamaica	-0.074666	Jamaica	-0.1216701	Russia	-0.1595116
Argentina	-0.0483719	Argentina	-0.1200658	Jamaica	-0.1551836
Romania	0.0218742	Romania	-0.1116163	Bulgaria	-0.0963784
Portugal	0.0251189	Bulgaria	-0.0841647	Romania	-0.0884699
Hungary	0.1940508	Chile	0.1344535	Chile	0.0975344
Chile	0.2369838	Hungary	0.1554265	Hungary	0.1492833
Poland	0.241313	Poland	0.2142966	Poland	0.2375489
Greece	0.4084024	Luxembourg	0.3695384	New Zealand	0.4038302
Luxembourg	0.4154634	Greece	0.4035965	Greece	0.4341967
United Kingdom	0.5539948	Czech Republic	0.5259696	Luxembourg	0.4717661
South Korea	0.612106	Switzerland	0.5577995	Czech Republic	0.5339431
Switzerland	0.6167299	Netherlands	0.5872815	Norway	0.5762146
Germany	0.6256459	New Zealand	0.6223932	Australia	0.6083979
France	0.6785437	Norway	0.6465203	Netherlands	0.6643482
Czech Republic	0.6935099	Australia	0.6520376	Sweden	0.7021803
Netherlands	0.6992378	Sweden	0.7050226	France	0.7534097
Norway	0.7722704	France	0.7054504	South Korea	0.8371592
Australia	0.7779676	South Korea	0.710041	Canada	0.8716802
Canada	0.8236649	United Kingdom	0.8367925	Switzerland	0.9344751
New Zealand	0.82897	Germany	0.8401804	Germany	0.9529409
Sweden	0.8888561	Canada	0.8439423	United Kingdom	0.9742275

Japan	1.030486	United States	1.170183	Japan	1.068849
United States	1.073756	Japan	1.18596	United States	1.233742

Source: Own elaboration

Appendix 3 - Residual endogeneity test for the results of table 1

Variables \ Model	Column 1	Column 2
lnfk	0.000	0.000
	(1)	(1)
lnhumancap	0.000	0.000
	(1)	(1)
lnoverall	0.000	-
	(1)	
Constant	0.000	0.000
	(1)	(1)
Observations	117	117
R^2	0.000	0.000

*** p <0.01, ** p <0.05, * p <0.1

† Standard errors in parentheses

Source: Own elaboration

Appendix 4 – The Extended Lucas Model

$$y = Ak_t^{\beta}(u_t h_t I_t)^{1-\beta} \quad (1)$$

$I_t = Je^{nt+\sum_{\tau=1}^{t}\varepsilon_\tau}$ given that $\sum_{\tau=1}^{t}\varepsilon_\tau = 0$, it has been $I_t = Je^{nt}$ (2)

$$y = Ak_t^{\beta}(u_t h_t J e^{nt})^{1-\beta} \quad (2')$$

$$\dot{h} = \theta h(1-u_t) \quad (3)$$

$$\dot{k} = Ak_t^{\beta}(u_t h_t I_t)^{1-\beta} - C \quad (4)$$

$$U(Ct) = \int_0^{\infty} e^{-\rho t}\frac{c_t^{1-\sigma}}{1-\sigma} \quad (5)$$

applying the Hamiltonian function

$$L = \int_1^{\infty} e^{-\rho t}\frac{c_t^{1-\sigma}}{1-\sigma} + V\left\{Ak_t^{\beta}(u_t h_t J e^{nt})^{1-\beta} - C\right\} + \lambda\{\theta h(1-u_t)\} \quad (5')$$

FOC

$$\frac{\partial L}{\partial u} = V\left\{Ak_t^{\beta}(1-\beta)(h_t J e^{nt})^{1-\beta}u_t^{-\beta}\right\} - \lambda\theta h = 0 \quad (6)$$

$$\frac{\partial L}{\partial c} = e^{-\rho t}c_t^{-\sigma} - V = 0 \quad (7)$$

$$\frac{\partial L}{\partial k} = V\left\{\beta A k_t^{\beta-1}(u_t h_t J e^{nt})^{1-\beta}\right\}$$ (8)

$$\frac{\partial L}{\partial h} = V\left\{(1-\beta)A k_t^{\beta}(u_t J e^{nt})^{1-\beta} h_t^{-\beta}\right\} + \lambda\{\theta(1-u_t)\}$$ (9)

transversality condition

$$\frac{\partial L}{\partial k} = -\dot{V}$$ (10)

$$\frac{\partial L}{\partial h} = -\dot{\lambda}$$ (11)

$$\frac{\partial L}{\partial u} = 0$$ (12)

(8) equals to (10)

$$V\left\{\beta A k_t^{\beta-1}(u_t h_t J e^{nt})^{1-\beta}\right\} = -\dot{V}$$

$$\frac{\dot{V}}{V} = -\left\{\beta A k_t^{\beta-1}(u_t h_t J e^{H_0 nt})^{1-\beta}\right\} \quad (13)$$

(9) equals to (11)

$$V\left\{(1-\beta)A k_t^{\beta}(u_t J e^{nt})^{1-\beta} h_t^{-\beta}\right\} + \lambda\{\theta(1-u_t)\} = -\dot{\lambda}$$ (14)

(12) equals to (6)

$$\left\{A k_t^{\beta}(1-\beta)(h_t J e^{nt})^{1-\beta} u_t^{-\beta}\right\} - \lambda\theta h = -\dot{\lambda} \quad (15)$$

log-linearizing (7)

$$-\rho t lne - \sigma lnct - lnV = 0$$

Deriving from t

$$-\rho - \sigma\frac{\dot{c}}{c} - \frac{\dot{V}}{V} = 0$$

$$\frac{\dot{c}}{c} = \frac{-\frac{\dot{V}}{V} - \rho}{\sigma}$$ (16)

Substituting (13) into (16),

$$\frac{\dot{c}}{c} = \frac{\beta A k_t^{\beta-1} u_t^{1-\beta} h_t^{1-\beta} J^{1-\beta} e^{nt(1-\beta)} - \rho}{\sigma} = \gamma$$ (17)

Dividing (4) by k_t

$$\frac{\dot{k}}{k} = Ak_t^{-(1-\beta)}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)} - \frac{c_t}{k_t} \quad (18)$$

In 17, if we multiply γ by σ, adding ρ and divide everything by β, we have M:

$$M = \frac{\gamma\sigma+\rho}{\beta} \quad (18')$$

Performing the procedure both sides of (17)

$$M = \frac{\gamma\sigma+\rho}{\beta} = \frac{\frac{\beta Ak_t^{\beta-1}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)}\sigma - \rho\sigma}{\sigma} + \rho}{\beta}$$

simplifying

$$M = \frac{\gamma\sigma+\rho}{\beta} = Ak_t^{\beta-1}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)} \quad (19)$$

Note that equation (19) is equal to the first term on the right-hand side of equation (18)

Substituting (19) into (18)

$$\frac{\dot{k}}{k} = \frac{\gamma\sigma+\rho}{\beta} - \frac{c_t}{k_t} = \gamma_k \quad (20)$$

log-linearizing (20)

$$ln\gamma + ln\sigma - ln\beta + ln\rho - ln\beta - lnc_t + lnk_t = ln\gamma_k$$

Deriving from t, we have:

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \gamma = \gamma_k \quad (21)$$

From (3), we have to

$$\frac{\dot{h}}{h} = \theta(1-u) = \gamma_h \quad (21')$$

From (19)

$$\frac{\gamma\sigma+\rho}{\beta} = Ak_t^{\beta-1}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)} \quad (22)$$

$$\frac{\gamma\sigma+\rho}{A\beta} = k_t^{\beta-1}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)} \quad (23)$$

log-linearizing (23)

$$\begin{aligned} ln\gamma + ln\sigma - lnA - lnB + ln\rho - lnA - lnB \\ = (\beta-1)lnk_t + (1-\beta)lnu_t + (1-\beta)lnh_t + (1-\beta)lnJ + nt(1-\beta) \end{aligned}$$

Deriving from t

$$0 = -(1-\beta)\frac{\dot{k}}{k} + (1-\beta)\frac{\dot{h}}{h} + n(1-\beta)$$
(24)

Isolating (24) to $\frac{\dot{h}}{h}$

$$\frac{\dot{h}}{h} = \frac{(1-\beta)}{(1-\beta)}\frac{\dot{k}}{k} - \frac{(1-\beta)n}{(1-\beta)} = \frac{(1-\beta)}{(1-\beta)}\gamma - \frac{(1-\beta)n}{(1-\beta)} = \gamma_h = \gamma - n$$
(25)

Dividing (6) by λ

$$\frac{V}{\lambda}\left\{Ak_t^{\beta}(1-\beta)(h_t J e^{nt})^{1-\beta}u_t^{-\beta}\right\} - \theta h = 0$$

Isolating to $\frac{V}{\lambda}$,

$$\frac{V}{\lambda} = \frac{\theta h}{Ak_t^{\beta}(1-\beta)(h_t J e^{nt})^{1-\beta}u_t^{-\beta}}$$

$$\frac{V}{\lambda} = \frac{\theta}{Ak_t^{\beta}(1-\beta)u_t^{-\beta}h_t^{-\beta}J^{1-\beta}e^{nt(1-\beta)}}$$
(26)

log-linearizing (26)

$$lnV - ln\lambda = ln\theta - lnA - \beta lnk_t - \ln(1-\beta) + \beta lnu_t + (\beta)lnh_t - nt(1-\beta) - (1-\beta)lnJ$$

Deriving from t

$$\frac{\dot{V}}{V} - \frac{\dot{\lambda}}{\lambda} = -\frac{\beta\dot{k}}{k} + (\beta)\frac{\dot{h}}{h} - n(1-\beta)$$

(Note: this result was obtained due to the fact that $\frac{\dot{u}}{u}$=0, which will be proved in equation (34)

Isolating to $\frac{\dot{\lambda}}{\lambda}$

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{V}}{V} + \frac{\beta\dot{k}}{k} - (\beta)\frac{\dot{h}}{h} + n(1-\beta) = \frac{\dot{V}}{V} + \beta\gamma - (\beta)\gamma_h + n(1-\beta)$$
(27)

From (13), we can see that $\frac{\dot{V}}{V} = -(\gamma\sigma + \rho)$
(27')

*Prooving (27'): from (17) we have

$$\frac{\beta Ak_t^{\beta-1}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)} - \rho}{\sigma} = \gamma$$

Multiplying γ by σ, adding ρ and multiplying all by -1, we have

$$-\left[\frac{\beta Ak_t^{\beta-1}u_t^{1-\beta}h_t^{1-\beta}J^{1-\beta}e^{nt(1-\beta)}\sigma - \sigma\rho}{\sigma} + \rho\right] = -(\gamma\sigma + \rho)$$

Simplifying

$-\beta A k_t^{\beta-1} u_t^{1-\beta} h_t^{1-\beta} J^{1-\beta} e^{nt(1-\beta)} = -(\gamma\sigma + \rho)$

Note that the left side of the equation is exactly equal to equation (13), so

$-\beta A k_t^{\beta-1} u_t^{1-\beta} h_t^{1-\beta} J^{1-\beta} e^{nt(1-\beta)} = -(\gamma\sigma + \rho) = \frac{\dot{V}}{V}$ (28)

*end of the proof

To find $\frac{\dot{\lambda}}{\lambda}$, we can divide the two sides of (14) by λ

$$\frac{V}{\lambda}\left\{(1-\beta)A k_t^{\beta}(u_t J e^{nt})^{1-\beta} h_t^{-\beta}\right\} + \{\theta(1-u_t)\} = -\frac{\dot{\lambda}}{\lambda}$$

Isolating to $\frac{\dot{\lambda}}{\lambda}$

$-\frac{V}{\lambda}\left\{(1-\beta)A k_t^{\beta}(u_t J e^{nt})^{1-\beta} h_t^{-\beta}\right\} - \{\theta(1-u_t)\} = \frac{\dot{\lambda}}{\lambda}$ (29)

Replacing (26) into (29)

$$\frac{\dot{\lambda}}{\lambda} = -\frac{\theta}{A k_t^{\beta}(1-\beta) u_t^{-\beta} h_t^{-\beta} J^{1-\beta} e^{nt(1-\beta)}}\left\{(1-\beta)A k_t^{\beta}(u_t J e^{nt})^{1-\beta} h_t^{-\beta}\right\} - \{\theta(1-u_t)\}$$

Simplifying

$\frac{\dot{\lambda}}{\lambda} = -\theta u_t - \theta + \theta u_t = -\theta$ (30)

From (27), isolating for $\frac{\dot{V}}{V}$, it has been

$$\frac{\dot{V}}{V} = \frac{\dot{\lambda}}{\lambda} - \beta\gamma + (\beta)\gamma_h - n(1-\beta)$$

Replacing (30)

$$\frac{\dot{V}}{V} = -\theta - \beta\gamma + (\beta)\gamma_h - n(1-\beta)$$

Replacing (25)

$$\frac{\dot{V}}{V} = -\theta - \beta\gamma + (\beta)[\gamma - n] - n(1-\beta)$$

Simplifying

$\frac{\dot{V}}{V} = -\theta - n$ (31)

Substituting (31) into (16):

$\frac{\dot{c}}{c} = \frac{\theta + H_0 n - \rho}{\sigma} = \gamma = \frac{\theta - \rho}{\sigma} + \frac{n}{\sigma}$ (32)

From (21) and (32):

$\frac{\dot{k}}{k} = \gamma = \frac{\theta-\rho}{\sigma} + \frac{n}{\sigma}$ (33)

**Proving that $\frac{\dot{u}}{u} = 0$,

From 3, we have to

$$(1-u_t) = \frac{\dot{h}}{h\theta}$$

Replacing (25):

$$(1-u_t) = \frac{\gamma}{\theta} - \frac{n}{\theta}$$

$$u_t = 1 - \frac{\gamma}{\theta} + \frac{n}{\theta}$$

log-linearizing both sides

$$lnu_t = ln1 + ln\theta - ln\gamma + lnn + ln - ln\theta$$

Deriving from t

$\frac{\dot{u}}{u} = 0$ (34)

*end of the proof

From 25 and 32, we have

$\gamma_h = \frac{\theta+n \ -\rho}{\sigma} - n = \frac{\theta}{\sigma} + \frac{n}{\sigma} - \frac{\rho}{\sigma} - n = \frac{(\theta-\rho)}{\sigma} + n(\frac{1}{\sigma} - 1)$
(35)

log-linearizing 2', and deriving in relation to time, we have:

$$\frac{\dot{y}}{y} = \beta\frac{\dot{k}}{k} + (1-\beta)\frac{\dot{h}}{h} + (1-\beta)n$$

$\frac{\dot{y}}{y} = \beta\left[\frac{\theta-\rho}{\sigma} + \frac{n}{\sigma}\right] + \left[(1-\beta)\left\{\frac{(\theta-\rho)}{\sigma} + n(\frac{1}{\sigma} - 1)\right\}\right] + (1-\beta)n$
(36)